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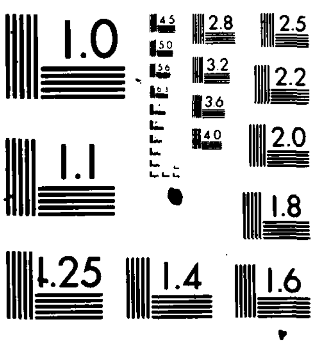

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WOLFQP, WOLFE'S METHOD FOR QUADRATIC PROGRAMMING

BY JOHN W. WINGATE,
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RESEARCH AND TECHNOLOGY DEPARTMENT

17 MARCH 1982

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC TR 82-30	2. GOVT ACCESSION NO. AD-A116288	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) WOLFQP, WOLFE'S METHOD FOR QUADRATIC PROGRAMMING		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John W. Wingate Jeffrey S. Youngs		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center (Code R44) White Oak Silver Spring, MD 20910		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 65856N,R0128-SB,R0128-SB, 2R44EA501
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 17 March 1982 13. NUMBER OF PAGES 94
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Quadratic Programming; Linear Programming; Constrained Optimization; Wolfe's Method

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report documents a FORTRAN routine WOLFQP for solving quadratic programming problems. Theory, usage notes and examples are included in the report.

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FOREWORD

This report documents a FORTRAN routine WOLFQP for solving quadratic programming problems. Theory, usage notes and examples are included in the report.

Ira M Blatstein

IRA M. BLATSTEIN
By direction

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CHAPTER 1

WOLFE'S METHOD FOR QUADRATIC PROGRAMMING

Wolfe's quadratic programmming method[1] is based on solving a system of linear relations subject to complementarity conditions. The linear system (without the complementarity conditions) can be solved by the simplex method of linear programming. The pivoting rules of the simplex method can be restricted so that the complementarity conditions are met. When the quadratic form in the objective function is positive definite (for a minimization problem) the pivoting restrictions do not prevent the simplex method from solving the linear system.

The linear system is based on the Karush-Kuhn-Tucker conditions (or Lagrange multiplier rule) for the problem to be solved. Let this problem be:

$$\text{minimize } 1/2 \begin{bmatrix} x_1^T & x_2^T \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} p_1^T & p_2^T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + C$$

subject to the constraints

$$x_1 \geq 0 \qquad (n_1 \text{ variables}),$$

$$x_2 \text{ unconstrained } (n_2 \text{ variables}),$$

$$\begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq b_1 \; (m_1 \text{ constraints}),$$

$$\begin{bmatrix} A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = b_2 \; (m_2 \text{ constraints}).$$

The constant term C is included for evaluation purposes only, it does not affect the minimizing point. If there are no unconstrained variables ($n_2 = 0$)

the matrix $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}$ reduces to Q_{11} and $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ reduces to $\begin{bmatrix} A_{11} \\ A_{21} \end{bmatrix}$

Similar interpretations apply when $n_1 = 0$, $m_1 = 0$ or $m_2 = 0$. Inequalities apply componentwise; i.e., $x_1 \geq 0$ is equivalent to $x_{1j} \geq 0$, $j = 1,\dots,n_1$. Q is positive definite.

Introduce Lagrange multiplier vectors κ_1 for the constraint $x_1 \geq 0$ and λ_1 and λ_2 for the other constraints, and form the Lagrangian

$$\begin{aligned} L = 1/2 \begin{bmatrix} x_1^T & x_2^T \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} p_1^T & p_2^T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + C \\ + \kappa_1^T \; x_1 \\ + \lambda_1^T \left(\begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - b_1 \right) \\ + \lambda_2^T \left(\begin{bmatrix} A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - b_2 \right) \end{aligned}$$

from which, by the Lagrange multiplier rule for inequality constrained problems, we get the linear conditions

$$\kappa_1 + \begin{bmatrix} Q_{11} & Q_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} A_{11}^T & A_{21}^T \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} + p_1 = 0,$$

$$\begin{bmatrix} Q_{12} & Q_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} A_{12}^T & A_{22}^T \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} + p_2 = 0,$$

along with the original constraints

$$x_1 \geq 0,$$

$$\begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq b_1,$$

$$\begin{bmatrix} A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = b_1,$$

the sign constraints on the multipliers for the inequalities

$$\kappa_1 \leq 0,$$

$$\lambda_1 \geq 0,$$

and the complementary slackness condition

$$\kappa_1^T \; x_1 + \lambda_1^T \left(\begin{bmatrix} A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - b_1 \right) + \lambda_2^T \left(\begin{bmatrix} A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - b_2 \right) = 0.$$

Since each of the $n_1 + m_1 + m_2$ terms in this sum is nonpositive by the constraints, each must individually vanish:

$$\kappa_{1j} = 0 \text{ or } x_{1j} = 0, \; j = 1, \ldots, n_1,$$

etc.

An alternate phrasing of this system of linear inequalities and equations can be made if we introduce auxiliary vector variables κ_2, S_1 and S_2.

Find a solution to:

$$\begin{bmatrix} I & 0 & 0 & 0 & Q_{11} & Q_{12} & A_{11}^T & A_{21}^T \\ 0 & I & 0 & 0 & Q_{12}^T & Q_{22} & A_{12}^T & A_{22}^T \\ 0 & 0 & I & 0 & A_{11} & A_{12} & 0 & 0 \\ 0 & 0 & 0 & I & A_{21} & A_{22} & 0 & 0 \end{bmatrix} \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ S_1 \\ S_2 \\ x_1 \\ x_2 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} -p_1 \\ -p_2 \\ b_1 \\ b_2 \end{bmatrix} \tag{1}$$

$$\kappa_1 \leq 0, \ \kappa_2 = 0, \ S_1 \geq 0, \ S_2 = 0, \ x_1 \geq 0, \ \lambda_1 \geq 0 \ . \tag{2}$$

and

$$\kappa_1^T \ x_1 = 0, \ S_1^T \ \lambda_1 = 0 \ . \tag{3}$$

(The complementary slackness conditions (3) again require each term in the scalar products $\kappa_1^T \ x_1$ and $S_1^T \ \lambda_1$ to vanish.) If we ignore the complementary slackness conditions, we can solve (1) and (2) by minimizing a penalty function

$$P = \sum_{j=1}^{n_1} \max\{0,\kappa_{1j}\} + \sum_{j=1}^{n_2} |\kappa_{2j}| + \sum_{j=1}^{m_1} \max\{-S_{1j},0\} + \sum_{j=1}^{m_2} |S_{2j}|$$

$$+ \sum_{j=1}^{n_1} \max\{-x_{1j},0\} + \sum_{j=1}^{m_1} \max\{-\lambda_{1j},0\}$$

Subject to (1) (and no sign constraints). The minimum value of P is 0 if and only if (1) and (2) are consistent. Of course, it is not necessary to weight all the sign constraint violations equally. For example, we could replace $|S_{2j}|$ by $\max\{-a \ S_{2j}, \ b \ S_{2j}\}$ where a and b are positive. Changing the weights in this manner does not affect the equivalence of the consistency of (1) and (2) to the minimum being zero. A slightly different procedure is normally used (in a simplex method Phase I). The identity matrix in the first $(n_1 + n_2 + m_1 + m_2)$ columns is used as an initial basis for the primal simplex method, any constraint violations resulting therefrom are penalized, and sign constraints are retained in such a way that the penalty function is linear. For example, if $b_{2j} > 0$, $S_{2j} = b_{2j}$ in the initial basic solution. The term $\cdots + S_{2j} + \cdots$ appears in the penalty and the restriction $S_{2j} \geq 0$ is imposed; S_{2j} is termed an "artificial variable" in this case. (If b_{2j} were negative, the penalty would be $-S_{2j}$ and S_{2j} kept nonpositive.) For S_1 the treatment is slightly different. If $b_{1j} \geq 0$, set $S_{1j} = b_{1j}$, with no penalty. If $b_{1j} < 0$, divide S_{1j} into the sum of a slack variable S_{1j} (unpenalized, nonnegative) and an artificial variable $S_{1j}{}^a$ (penalized, nonpositive), set to b_{1j} initially. This procedure is, however, equivalent to changing the weights in P. Using $b_{2j} > 0$ again as an example, we could change the weighting so that the penalty on S_{2j} changes from $|S_{2j}|$ to $\max\{(-\infty)S_{2j}, S_{2j}\}$ and in practice we could replace ∞ by a suitable large positive quantity M.

It is not actually necessary to use a linear cost functional. With a little care in programming, the usual simplex method can handle cost functionals like P. The linear programming code LINOPT[2] uses the dual simplex method to solve problems of maximizing a linear functional subject to upper and lower bounds on all variables and constrained quantities. The problem of minimizing P subject to (1) is the dual of such a problem, which could be solved using LINOPT, with the effect of minimizing P subject to (1) by the primal simplex method.

Let us now consider the complementary slackness conditions (3). In terms of the primal simplex method, they can be enforced by ensuring that S_{1j} and its complementary variable λ_{1j} (κ_{1j} and x_{1j}) are not simultaneously basic. This is an easily enforced pivoting restriction. Unfortunately it is too strong to be applied to minimizing P subject to (1) directly: the algorithm may terminate with $P > 0$ even though (1), (2) and (3) are consistent. We get around this difficulty by using a phase I/phase II procedure. Suppose we split κ_1 into a multiplier κ_1^m, subject to complementary pivoting restrictions, and an artificial variable κ_1^a, not so restricted: $\kappa_1 = \kappa_1^m + \kappa_1^a$. Since κ_{1j}^m and κ_{1j}^a cannot both be basic, one of them is always zero and nonbasic.

We can let κ_{1j} be whichever of them is basic (if one is) and keep track of whether this is κ_{1j}^a or κ_{1j}^m.

Suppose now that we have a basic feasible solution to the constraints on x. Then

$$\begin{bmatrix} I & 0 & A_{11} & A_{12} \\ 0 & I & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$S_1 \geq 0, \; S_2 = 0, \; x_1 \geq 0$$

Extend this basis to the constraint matrix for (1) by making κ_1^a and κ_2 basic. If we now minimize $P' = \sum_{j=1}^{n_1} |\kappa_{1j}^a| + \sum_{j=1}^{n_2} |\kappa_{2j}|$ subject to (1) and

$\kappa_1 = \kappa_1^a + \kappa_1^m$, $\kappa_1^m \leq 0$, $x_1 \geq 0$, $S_1 \geq 0$, $S_2 = 0$, $\lambda_1 \geq 0$ and the complementary pivoting restriction, the minimum will be zero, and we will have solved (1), (2) and (3). (The proof of this fact requires the positive definiteness of Q.)

If we wish to enforce the sign constraints by penalty terms added to P' these terms must be so heavily weighted that no change of basis that reduces P' can introduce sign constraint violations.

The phase I is to find the initial basic feasible solution. It is not necessary to work on a smaller problem using only the [I A] part of the full coefficient matrix $\begin{bmatrix} I & 0 & Q & A^T \\ 0 & I & A & 0 \end{bmatrix}$. If we allow κ_1 and κ_2 to be basic and unconstrained in phase I, they will be basic throughout phase I and the initial basis inverse for Phase II will be the final basis inverse for phase I.

Phase I problem:

$$\text{minimize } P_I = M \left\{ \sum_{j=1}^{m_1} \max\{-S_{1j},0\} + \sum_{j=1}^{m_2} |S_{2j}| + \sum_{j=1}^{n_1} \max\{-x_{1j},0\} + \sum_{j=1}^{m_1} \max\{-\lambda_{1j},0\} \right\}$$

subject to:

$$\begin{bmatrix} I & 0 & 0 & 0 & Q_{11} & Q_{12} & A_{11}^T & A_{21}^T \\ 0 & I & 0 & 0 & Q_{12}^T & Q_{22} & A_{12}^T & A_{22}^T \\ 0 & 0 & I & 0 & A_{11} & A_{12} & 0 & 0 \\ 0 & 0 & 0 & I & A_{21} & A_{22} & 0 & 0 \end{bmatrix} \begin{bmatrix} \kappa_1 \\ \kappa_2 \\ S_1 \\ S_2 \\ x_1 \\ x_2 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} -p_1 \\ -p_2 \\ b_1 \\ b_2 \end{bmatrix}$$

where M >> 1. Since M scales out of the problem it is not actually needed here. It is included as part of the setup for phase II. A similar comment applied to the penalty on λ_1 which remains nonbasic and zero throughout phase I (and could therefore be ignored). Starting basic variables are κ_1, κ_2, S_1 and S_2. The program actually sets up and solves the dual to this problem. Let k_1 be a vector of variables dual to κ_1, k_2 dual to κ_2,σ_1 dual to S_1, σ_2 dual to S_2, ξ_1 dual to x_1, ξ_2 dual to x_2, ℓ_1 dual to λ_1 and ℓ_2 dual to λ_2. For both phases the dual variables are related by the transpose of the coefficient matrix:

$$\begin{bmatrix} k_1 \\ k_2 \\ \sigma_1 \\ \sigma_2 \\ \xi_1 \\ \xi_2 \\ \ell_1 \\ \ell_2 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ Q_{11} & Q_{12} & A_{11}^T & A_{21}^T \\ Q_{12}^T & Q_{22} & A_{12}^T & A_{22}^T \\ A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ \sigma_1 \\ \sigma_2 \end{bmatrix} \qquad (4)$$

The dual phase I problem is to maximize $z = [-p_1{}^T \; -p_2{}^T \; b_1{}^T \; b_2]\begin{bmatrix}k_1\\ k_2\\ \sigma_1\\ \sigma_2\end{bmatrix}$ subject to the bounds:

$$k_1 = 0,\; k_2 = 0,\; -M \le \sigma_1 \le 0,\; -M \le \sigma_2 \le M$$

$$-M \le \xi_1 \le 0,\; \xi_2 = 0,\; -M \le \ell_1 \le 0,\; \ell_2 = 0$$

(The bounds on ℓ_1 and ℓ_2 are obviously always satisfied in phase I.) The objective function is defined by the right-hand side of (1). The bounds are defined by the penalties in P_I: the bounds for a dual variable deriving from the penalties on the corresponding variable. Violation of nonzero bounds in phases I and II are ignored: they can be made to go away by increasing the penalty on the appropriate variable and we could have set up the penalty function in this way in the first place. Furthermore, ignoring violation of nonzero bounds yields a more efficient procedure.

Assuming that phase I ends with feasibility shown, we start phase II by changing the penalty from P_I to

$$P_{II} = P_I + \sum_{j=1}^{n_1} |\kappa_{1j}^a| + \sum_{j=1}^{n_2} |\kappa_{2j}| + \sum_{j=1}^{n_1} \max\{0, \kappa_{1j}^m\},$$

or equivalently, changing the bounds on k_1 and k_2. For k_2 this is easy: $-1 \le k_2 \le 1$, but for k_1, setting the bounds is complicated by the implicit handling of the split of κ_1 into $\kappa_1{}^m + \kappa_1{}^a$. Let us start by making each artificial variable κ_{1j}^a basic, and treat k_{1j} as its dual, with bounds $-1 \le k_{1j} \le 1$. Then k_{1j} will initially be either +1 or -1 (depending on the sign of κ_{1j}^a). Ordinarily, without the pivoting restriction, if $k_{1j} = k_{1j}^a = -1$, the lower bound (zero) on $k_{1j}^m = k_{1j}^a = k_{1j}$ would be violated, and we could pivot k_{1j}^m into the basis with value zero, with no change in the basis inverse matrix. With the pivoting restriction, we can still do this if ξ_{1j} is not basic (and zero--but it will be zero if basic because we ignore violation of nonzero bounds). Since we want to reduce artificial variables to zero, we might as well do so at the start. Thus the initial setup of bounds on k_1 are: $r_j \le k_{1j} \le 1$

where $r_j = -1$ if ξ_{1j} is basic at the end of phase I,

$r_j = 0$ if ξ_{1j} is nonbasic at the end of phase I.

In proceeding through phaseII, we should interpret k_{1j} as dual to k^m_{1j} as soon as possible. Thus r_j should be set to zero whenever ξ_{1j} leaves the basis, not only the lower bound, but k_{1j} itself should be reset.

Since each variable in ℓ_1 is nonbasic at the *start* of phase II, the complementarity restrictions on ℓ_1 and ℓ_1 cause no problems in setting up for phase II.

Comments in the modified LINOPT code referring to pivoting restrictions are given in terms appropriate to the dual problem (4) with phase II bounds. A further thing to note is that the negation of p_1 and p_2 in the objective row

$$[-p_1^T \quad -p_2^T \quad b_1^T \quad b_2^T]$$

is handled internally, so that this vector should be passed as

$$[\ p_1^T \quad p_2^T \quad b_1^T \quad b_2^T].$$

CHAPTER 2
USER'S NOTES

WOLFQP is a FORTRAN Language computer code written to solve problem (1) with the method described in the previous section. The code is a modified version of LINOPT which takes advantage of the structure in the Quadratic Programming problem to reduce the storage required. Also the user is only required to input the QP problem, it is translated by WOLFQP into the Linear Programming Problem.

Communication with the calling program is accomplished through the calling statement. The formal parameters are described thoroughly in the internal documentation (See Appendix A). This section therefore will only attempt to clarify some of the murkier details.

The scratch array SCR must be dimensioned at least $N^2 + 13N + 6$ ($N = N_1 + N_2 + M_1 + M_2$; where N_1 = Number of sign constrained variables, N_2 = Number of sign unconstrained variables, M_1 = Number of unequality constraints, M_2 = Number of equality constraints). Failure to do this will cause memory to be overwritten and result in indeterminate output. The scratch array is divided into the internal arrays required of LINOPT. The correlation between these internal variables and positions in SCR are detailed in the internal documentation (See Appendix A). For detailed descriptions of the internal variables see reference [1].

It is noted that the unknown vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in problem (1) has sign constrained x_1 and sign unconstrained x_2 variables. For ease of notation the constrained and free variables are grouped separately. WOLFQP does not require the user to so order the variables. The integer vector IND allows the user to permute the variables without permuting the Q, A1 and A2 matrices. This allows the user to form the matrices in a convenient manner.

The First N_1 locations of IND contain the indices for which there are sign constraints. Locations $N_1 + 1 \ldots, N_1 + N_2$ contain the indices for the free variables.

Also it may be inconvenient to reform the constraint matrices for a call to WOLFQP. As an example the user may wish to call WOLFQP with only some of the constraint rows active. Locations $N_1 + N_2 + 1 \ldots N_1 + N_2 + M_1$ of IND contain the indices of the active rows, if any, in the inequality constraint matrix A1. In the same manner locations $N_1 + N_2 + M_1 + 1 \ldots N_1 + N_2 + M_1 + M_2$ of IND contain the indices of the active rows, if any, in A2; the equality constraint matrix.

If the value of the objective function is desired the user should set the formal parameter VALUE to a non-zero value. This results in the calculation of the objectives function value at the optimal solution. The Scalar C is ignored except in this calculation.

Two inputs which control the algorithm are ITMAX and NINVT, corresponding to IPASS (2) and IPASS(8) respectively. ITMAX is the iteration limit. Upon completion of ITMAX iterations control is returned to the calling program with IERR (IPASS (4)) set to 2. All internal storage has been saved. If desired WOLFQP may be recalled and computation continued by calling WOLFQP with FLAG set to true. No other change is necessary or advised.

On output the optimal distribution is placed in the array Y. The value is calculated, if desired. The iteration number and error indicator are stored in the appropriate positions of IPASS (See Appendix A). Also if the user desires to look at the internal variables SCR contains these values.

Other error conditions (IERR = 1,3,4+) must be corrected before re-calling WOLFQP. For these conditions the tableau must be re-initialized. (Flag set to False).

An upper limit on ITMAX should be between 5*N and 10*N. Some problems may take more; most should take fewer iterations. Practically ITMAX should be set so that an inordinate amount of CP time is not consumed before the problem formulation has been thoroughly checked.

NINVT controls the re-inversion of the tableau. Since succeeding inverses are formed as perturbations of previous inverses truncation error can accumulate in the inverse. To remedy this situation the inverse must be re-formed. Every NINVT iterations the inverse will be reformed. To avoid unnecessary calculations this should not occur too frequently. The recommended value is between 2 x N and 3 x N.

The final input controlling calculations is EPS1. This is used to control zero tolerance. Zero tolerance is important in two parts of the algorithm. Tableau entries which are less than EPS1 in magnitude are treated as zero.

Their values are considered to be truncation error. Thus for this purpose EPS1 is set to the truncation error of the machine.

The other spot where zero tolerance is involved is when constraint violations are checked. Constraint violations less than EPS1 are treated as zero. This is also to avoid truncation error but the user must be careful. If the problem is scaled such that entries in the tableau are less than EPS1 their entries will be ignored. For this purpose EPS1 must be on the order of the smallest entries in the tableau (Q, A_1 and A_2). The input value for EPS1 should be the minimum of these two values.

The major modification to LINOPT occurs in PIVROW. This subroutine chooses the incoming basic variable which is required to meet the complementary slackness condition. Flags contained in the internal array BASIC are used to facilitate this checking.

Each word in the BASIC array contains seven bits used as flags and if the complementary slackness condition applied to the variable, (a complementary index). Bits are used to reduce the storage requirement. The layout of each basic word is described in the internal documentation of SETQP. (See Appendix A).

The Bit operations OR, AND, SHIFT and COMPLEMENT are required to access and define the flag bits. These operations are described more thoroughly in machine dependencies (Appendix C).

The Flag Bits are also used in DSIMP. In DSIMP the basic or non-basic bit is set as appropriate and bounds are reset, if applicable, on variables leaving the basis.

CHAPTER 3

EXAMPLES

A short test program appears at the end of the source listing. (Appendix A). The problem is defined by the NAMELIST inputs. Mnemonic names are used in the NAMELIST which are then loaded into the IPASS array. The following examples list the NAMELIST inputs and the output resulting from a subsequent call to QPTAB.

Example 1.

This problem illustrates the basic use of the algorithm.

$$\min 1/2\ [x_1,\ x_2] \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [6\ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

s.t.

$x_1,\ x_2 \geq 0$

$x_1 \leq 2$

$x_2 \leq 1$

$x_1 + x_2 = 2$

This problem requires the following inputs

```
$IN
  EPS = 1.E-10,
  FLAG = .FALSE.,
  IND = 1,2,1,2,1,
  N = 5,
  NCON = 2,
  NUNC = 0,
   NINEQ = 2,
  NINV = 3000,
  ITMAX = 100,
  Q = 4., -2.,
  Q(1,2) = 2.,4.,
  A1 = 1., 0.,
  A1(1,2) = 0.,1.,
  A2(1,1) = 1.,
  A2(1,2) = 1.,
  OBJ = 6.,0.,2.,1.,2.,
  C = 0.,
  VALUE = 1.,
$END
```

output of QPTAB after call to WOLFQP

```
$OUT
N       =  5,
NCON    =  2,
NUNC    =  0,
NINEQ   =  2,
NEQ     =  1,
NINV    =  3000,
ITMAX   =  100,
ITER    =  2,
IERR    =  0,
C       =  0.0,
VALUE   =  .8E+01,
$END
       Q
  .4000E+01   -.2000E+01
 -.2000E+01    .4000E+01


       P
  .6000E+01   0.

   A1*X          B1              A1
  .1000E+01    .2000E+01    .1000E+01  0.
  .1000E+01    .1000E+01   0.           .1000E+01

   A2*X          B2              A2
  .2000E+01    .2000E+01    .1000E+01   .1000E+01


       X
  .1000E+01    .1000E+01
```

Basically the output reprints the inputs with the addition of the solution vector X = [1.,1.], the value of the function VALUE = 8., (VALUE on input was non-zero). The number of iterations required ITER = 2, the error indicator IERR = 0 indicating solution and each constraint is evaluated as a check. (See A1*X and A2*X columns.

Example 2.

This example illustrates how sign constraints can be written implicitly or explicitly. The inputs differ slightly. The explicit version requires more iterations.

$$\min 1/2\ [x_1, x_2, x_3, x_4] \begin{bmatrix} 1 & 0 & 6 & 1 \\ 0 & 4 & 6 & 4 \\ 6 & 6 & 61 & 12 \\ 1 & 4 & 12 & 100 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$+ \begin{bmatrix} 0, & 0, & 0, & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Inputs for Problem 1

```
$IN
  EPS = 1.E-10,
  FLAG = .FALSE.,
  OBJ = 10*0.,
     N = 4.,
  NCON = 4,
  NUNC = 0,
  NINEQ = 0,
  NINV = 2000,
  ITMAX = 100,
  Q(1,1) = 1.,0.,6.,1.,
  Q(1,2) = 0.,4.,6.,4.,
  Q(1,3) = 6.,6.,61.,12.,
  Q(1,4) = 1.,4.,12.,100.,
  OBJ = 0.,0.,0.,-1.,
  C = 0.,
  VALUE = 1.,
  IND = 1,2,3,4,
$END
```

One notes that the sign constraints are entered implicitly and are the only constraints.

Outputs:

```
$OUT
N      =  4,
NCON   =  4,
NUNC   =  0,
NINEQ  =  0,
NEQ    =  0,
NINV   =  2000,
ITMAX  =  100,
ITER   =  1,
IERR   =  0,
C      =  0.0,
VALUE  =  -.5E-02,
$END
        Q
  .1000E+01   0.          .6000E+01    .1000E+01
 0.            .4000E+01  .6000E+01    .4000E+01
  .6000E+01    .6000E+01  .6100E+02    .1200E+02
  .1000E+01    .4000E+01  .1200E+02    .1000E+03


        P
 0.           0.          0.          -.1000E+01

        X
 0.           0.          0.           .1000E-01
```

Note only 1 iteration is required.

The following inputs solve the same problem, but x_1 and x_2 are implicitly constrained. x_3 and x_4 have explicit constraints in the A1 matrix.

Inputs:

```
$IN
  EPS = 1.E-10,
  FLAG = .FALSE.,
  A1 = 300*0., A2 = 300*0., Q=900*0., OBJ = 30*0.,
  OBJ = 10*0,
     N = 6,
  NCON = 2,
  NUNC = 2,
  NINEQ = 2,
  NINV = 2000,
  ITMAX = 100,
  Q(1,1) = 1.,0.,6.,1.,
        Q(1,2) = 0.,4.,6.,4.,
        Q(1,3) = 6.,6.,61.,12.,
        Q(1,4) = 1.,4.,12.,100.,
        A1(1,1) = -1.,0.,0.,0.,
        A1(1,2) = 0.,-1.,0.,0.,
        A1(1,3) = 0.,0.,-1.,0.,
        A1(1,4) = 0.,0.,0.,-1.,
        OBJ = 0.,0.,0.,-1.,
        C = 0.,
        VALUE = 1.,
        IND = 1,2,3,4,3,4,
       $END
```

Output:

```
$OUT
N       =  6,
NCON    =  2,
NUNC    =  2,
NINEQ   =  2,
NEQ     =  0,
NINV    =  2000,
ITMAX   =  100,
ITER    =  3,
IERR    =  0,
C       =  0.0,
VALUE   =  -.5E-02,
$END

          Q
  .1000E+01      0.              .6000E+01     .1000E+01
 0.                .4000E+01     .6000E+01     .4000E+01
  .6000E+01        .6000E+01     .6100E+02     .1200E+02
  .1000E+01        .4000E+01     .1200E+02     .1000E+03

          P
 0.              0.              0.            -.1000E+01

    A1*X             B1              A1
 0.              0.              0.            0.          -.1000E+01   0.
 -.1000E-01      0.              0.            0.          0.           -.1000E+01

          X
 0.              0.              0.             .1000E-01
```

Note the solution is the same as the implicitly constrained example but this solution required 3 interations instead of 1 the problem has increased in size from N = 4 to N = 6 thus requiring additional storage.

If one wishes to continue in this vein, all sign constraints can be made explicit. The dimension of the problem becomes N = 8. The same solution now requires 7 iterations. The reader may verify this if desired.

Example 3

This is an example displaying the restart capabilities of the code. Since the Q matrix is the identity this is a constrained least squares problem of a particularly **simple type**. By following the directions in Appendix B the user can eliminate the need to store the identity matrix in memory.

This problem arises as a sub-problem of the feasible direction method. What is required is to find the direction of steepest descent subject to the binding constraints. The feasible direction method can at times require many constrained gradients to be calculated. The constraint matrix is usually full throughout the process but which constraints are binding depend on the given position where the constrained gradient is to be calculated.

This motivates the use of the linked list IND. First to indicate in the first N_1 positions the indices of the sign constrained variables. In the following N_2 positions the indices of sign unconstrained variables. Thus the variables need not be ordered constrained followed by unconstrained. The next M_1 positions of IND are filled with the row indices of the active inequality constraints and finally the last M_2 positions are filled by the indices of the active equality constraints.

Thus for the feasible direction method the full constraint matrix is formed once. Afterward only the row indices of the matrices must be manipulated to pass the current constraint set. Note the objective row containing the right hand side of the constraints must be reformed each time it changes.

The gradient projection problem can be stated:

$$\min_{\hat{g}} 1/2 < \hat{g} - g(x), \hat{g} - g(x) >$$

where $g(x)$ is the gradient at x and $\hat{g}$ is the constrained gradient.

Subject to

$$\hat{g}_i \geq 0 \text{ for all } x_i = 0$$

Maintaining x_i as non-negative

$$A_1 \hat{g} \leq 0$$

$$A_2 \hat{g} = 0$$

where for the gradient project problem A_1 and A_2 are the constraints matrices on K.

Inputs

```
$IN
  EPS = 1.E-10
  FLAG = .FALSE.,
  C = 0.,
  VALUE = 1.,
  N = 26,
  NCON = 12,
  NUNC = 8,
  NINEQ = 4,
  NINV = 3000,
  ITMAX = 100,
  Q = 900 * 0.,
  Q(1,1) = 1.,
  Q(2,2) = 1.,
  Q(3,3) = 1.,
  Q(4,4) = 1.,
  Q(5,5) = 1.,
  Q(6,6) = 1.,
  Q(7,7) = 1.,
  Q(8,8) = 1.,
  Q(9,9) = 1.,
  Q(10,10) = 1.,
  Q(11,11) = 1.,
  Q(12,12) = 1.,
  Q(13,13) = 1.,
  Q(14,14) = 1.,
  Q(15,15) = 1.,
  Q(16,16) = 1.,
  Q(17,17) = 1.,
  Q(18,18) = 1.,
  Q(19,19) = 1.,
  Q(20,20) = 1.,
  A1 = 300*0.,
  A1(1,1) = 1.,
  A1(1,2) = 1.,
  A1(1,3) = 1.,
  A1(1,4) = 0.,1.,
  A1(1,5) = 0.,1.,
  A1(1,6) = 0.,1.,
  A1(1,11) = 2*0.,1.,
  A1(1,12) = 2*0.,1.,
  A1(1,13) = 2*0.,1.,
  A1(1,14) = 3*0.,1.,
  A1(1,15) = 3*0.,1.,
 A1(1,16) = 3*0.,1.,
 A2(1,1) = 300*0.,
 A2(1,1) = 1.,
 A2(1,2) = 1.,
 A2(1,3) = 1.,
 A2(1,4) = 1.,
 A2(1,5) = 1.,
 A2(1,6) = 1.,
   A2(1,7) = 1.,
 A2(1,8) = 1.,
 A2(1,9) = 1.,
 A2(1,10) = 1.,
 A2(1,11) = 0.,1.,
 A2(1,12) = 0.,1.,
 A2(1,13) = 0.,1.,
 A2(1,14) = 0.,1.,
 A2(1,15) = 0.,1.,
 A2(1,16) = 0.,1.,
 A2(1,17) = 0.,1.,
 A2(1,18) = 0.,1.,
 A2(1,19) = 0.,1.,
 A2(1,20) = 0.,1.,
 OBJ = -1.,-.566,-.8187,-.9927,-.8187,-.8187,-.8047,-.7800,-.9643,
   -.7701,-.0659,-.0221,-.0130,-.0667,-.0158,-.0043,14*0.,
 IND = 2,3,5,6,8,10,11,12,13,18,19,20,1,4,7,9,14,15,16,17,1,2,3,4,1,2,
$END
```

Outputs:

```
$OUT
N       = 26,
NCON    = 12,
NUNC    = 8,
NINEQ   = 4,
NEQ     = 2,
NINV    = 3000,
ITMAX   = 100,
ITER    = 26,
IERR    = 0,
C       = 0.0,
VALUE   = -.74708433333331E-02,
$END
.1000E+01 0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        .1000E+01 0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        .1000E+01 0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        .1000E+01 0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        .1000E+01 0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        .1000E+01 0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        .1000E+01 0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        .1000E+01 0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        .1000E+01 0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        .1000E+01
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.

0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        0.
.1000E+01 0.        0.        0.        0.        0.        0.        0.        0.        0.
0.        .1000E+01 0.        0.        0.        0.        0.        0.        0.        0.
0.        0.        .1000E+01 0.        0.        0.        0.        0.        0.        0.
0.        0.        0.        .1000E+01 0.        0.        0.        0.        0.        0.
0.        0.        0.        0.        .1000E+01 0.        0.        0.        0.        0.
0.        0.        0.        0.        0.        .1000E+01 0.        0.        0.        0.
0.        0.        0.        0.        0.        0.        .1000E+01 0.        0.        0.
0.        0.        0.        0.        0.        0.        0.        .1000E+01 0.        0.
0.        0.        0.        0.        0.        0.        0.        0.        .1000E+01 0.
0.        0.        0.        0.        0.        0.        0.        0.        0.        .1000E+01
```

Outputs Continued:

```
      P

-.1000E+01   -.8566E+00   -.8187E+00   -.9927E+00   -.8187E+00   -.8187E+00   -.8047E+00   -.7800E+00   -.9643E+00   -.7701E+00
-.6590E-01   -.2210E-01   -.1300E-01   -.6670E-01   -.1580E-01   -.4300E-02   0.           0.           0.           0.

  A1*X         B1           A1

0.           0.           .1000E+01    .1000E+01    .1000E+01    0.           0.           0.           0.           0.
0.           0.           0.           0.           0.           .1000E+01    .1000E+01    .1000E+01    0.           0.
0.           0.           0.           0.           0.           0.           0.           0.           0.           0.
-.1110E-15   0.           0.           0.           0.           0.           0.           0.           0.           0.

                          0.           0.           0.           0.           0.           0.           0.           0.
                          0.           0.           0.           0.           0.           0.           0.           0.
                          0.           0.           .1000E+01    .1000E+01    .1000E+01    0.           0.           0.
                          0.           0.           0.           0.           0.           .1000E+01    .1000E+01    .1000E+01

                          0.           0.           0.           0.
                          0.           0.           0.           0.
                          0.           0.           0.           0.
                          0.           0.           0.           0.

  A2*X         B2           A2

0.           0.           .1000E+01    .1000E+01    .1000E+01    .1000E+01    .1000E+01    .1000E+01    .1000E+01    .1000E+01
-.1110E-15   0.           0.           0.           0.           0.           0.           0.           0.           0.

                          .1000E+01    .1000E+01    0.           0.           0.           0.           0.           0.
                          0.           0.           .1000E+01    .1000E+01    .1000E+01    .1000E+01    .1000E+01    .1000E+01

                          0.           0.           0.           0.
                          .1000F+01    .1000E+01    .1000E+01    .1000E+01

      X

0.           0.           0.           0.           0.           0.           -.7980E-01   0.           .7980E-01    0.
0.           0.           0.           .3777E-01    -.1317E-01   -.2463E-01   0.           0.           0.           0.
```

Note the constraint matrices. The unequality constraints are groups of three consecutive variables. The equality constraints are groups of ten consecutive variables. Also 26 iterations were required to determine the solution.

This next problem is identical to the previous but instead of using the permutation indice the matrices have been reformed. Sign constrained variables have been ordered first followed by sign unconstrained variables.

If this must be done many times for a given constraint tableau one can see a considerable work would be required. Also ITMAX has been set to 5.

Inputs:

```
$IN
  EPS = 1.E-10,
  FLAG = .FALSE.,
  IND = 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,1,2,3,4,1,2,
  C = 0.,
  VALUE = 1.,
  N = 26,
  NCON = 12,
  NUNC = 8,
  NINEQ = 4,
  NINV = 3000,
  ITMAX = 5,
  Q = 900 * 0.,
  Q(1,1) = 1.,
  Q(2,2) = 1.,
  Q(3,3) = 1.,
  Q(4,4) = 1.,
  Q(5,5) = 1.,
  Q(6,6) = 1.,
  Q(7,7) = 1.,
  Q(8,8) = 1.,
  Q(9,9) = 1.,
  Q(10,10) = 1.,
  Q(11,11) = 1.,
  Q(12,12) = 1.,
  Q(13,13) = 1.,
  Q(14,14) = 1.,
  Q(15,15) = 1.,
  Q(16,16) = 1.,
  Q(17,17) = 1.,
  Q(18,18) = 1.,
  Q(19,19) = 1.,
  Q(20,20) = 1.,
  A1 = 300*0.,
  A1(1,1) = 1.,
  A1(1,2) = 1.,
  A1(1,3) = 0.,1.,
  A1(1,4) = 0.,1.,
  A1(1,7) = 2*0.,1.,
  A1(1,8) = 2*0.,1.,
  A1(1,9) = 2*0.,1.,
  A1(1,13) = 1.,
  A1(1,14) = 0.,1.,
  A1(1,17) = 3*0.,1.,
  A1(1,18) = 3*0.,1.,
  A1(1,19) = 3*0.,0.,
  A2(1,1) = 300*0.,
  A2(1,1) = 1.,
  A2(1,2) = 1.,
  A2(1,3) = 1.,
  A2(1,4) = 1.,
  A2(1,5) = 1.,
  A2(1,6) = 1.,
     A2(1,7) = 0.,1.,
  A2(1,8) = 0.,1.,
  A2(1,9) = 0.,1.,
  A2(1,10) = 0.,1.,
  A2(1,11) = 0.,1.,
  A2(1,12) = 0.,1.,
  A2(1,13) = 1.,
  A2(1,14) = 1.,
  A2(1,15) = 1.,
  A2(1,16) = 1.,
  A2(1,17) = 0.,1.,
  A2(1,18) = 0.,1.,
  A2(1,19) = 0.,1.,
  A2(1,20) = 0.,1.,
  OBJ = -.8566,-.8187,-.8187,-.8187,-.78,-.7701,-.0659,-.0221,-.0130,
     3*0.,-1.,-.9927,-.8047,-.9643,-.0667,-.0158,-.0043,11*0.,
$END
```

Outputs:

```
 $OUT
 N       = 26,
 NCON    = 12,
 NUNC    = 8,
 NINEQ   = 4,
 NEQ     = 2,
 NINV    = 3000,
 ITMAX   = 5,
 ITER    = 5,
 IERR    = 2,
 C       = 0.0,
 VALUE   = .1E+01,
 $END
          Q
.1000E+01  0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         .1000E+01  0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         .1000E+01  0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         .1000E+01  0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         .1000E+01  0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         .1000E+01  0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         .1000E+01  0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         .1000E+01  0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         .1000E+01  0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         .1000E+01
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.

0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         0.
.1000E+01  0.         0.         0.         0.         0.         0.         0.         0.         0.
0.         .1000E+01  0.         0.         0.         0.         0.         0.         0.         0.
0.         0.         .1000E+01  0.         0.         0.         0.         0.         0.         0.
0.         0.         0.         .1000E+01  0.         0.         0.         0.         0.         0.
0.         0.         0.         0.         .1000E+01  0.         0.         0.         0.         0.
0.         0.         0.         0.         0.         .1000E+01  0.         0.         0.         0.
0.         0.         0.         0.         0.         0.         .1000E+01  0.         0.         0.
0.         0.         0.         0.         0.         0.         0.         .1000E+01  0.         0.
0.         0.         0.         0.         0.         0.         0.         0.         .1000E+01  0.
0.         0.         0.         0.         0.         0.         0.         0.         0.         .1000E+01
```

Output Continued:

```
       P

-.8566E+00  -.8187E+00  -.8187E+00  -.8187E+00  -.7800E+00  -.7701E+00  -.6590E-01  -.2210E-01  -.1300E-01  0.
0.          0.          -.1000E+01  -.9927E+00  -.8047E+00  -.9643E+00  -.6670E-01  -.1580E-01  -.4300E-02  0.

  A1*X         B1               A1

0.          0.           .1000E+01   .1000E+01  0.          0.          0.          0.          0.          0.
0.          0.          0.          0.           .1000E+01   .1000E+01  0.          0.          0.          0.
0.          0.          0.          0.          0.          0.          0.          0.           .1000E+01   .1000E+01
0.          0.          0.          0.          0.          0.          0.          0.          0.          0.

                        0.          0.          0.          0.           .1000E+01  0.          0.          0.
                        0.          0.          0.          0.          0.           .1000E+01  0.          0.
                         .1000E+01  0.          0.          0.          0.          0.          0.          0.
                        0.          0.          0.          0.          0.          0.          0.          0.

                        0.          0.          0.          0.
                        0.          0.          0.          0.
                        0.          0.          0.          0.
                         .1000E+01   .1000E+01   .1000E+01  0.

  A2*X         B2               A2

0.          0.           .1000E+01   .1000E+01   .1000E+01   .1000E+01   .1000E+01   .1000E+01  0.          0.
0.          0.          0.          0.          0.          0.          0.          0.           .1000E+01   .1000E+01

                        0.          0.          0.          0.           .1000E+01   .1000E+01   .1000E+01   .1000E+01
                         .1000E+01   .1000E+01   .1000E+01   .1000E+01  0.          0.          0.          0.

                        0.          0.          0.          0.
                         .1000E+01   .1000E+01   .1000E+01   .1000E+01

       X

        -R          -R          -R          -R          -R          -R          -R          -R          -R          -R
        -R          -R          -R          -R          -R          -R          -R          -R          -R          -R
```

Note IERR returns equal to 2 indicating the iteration limit has been reached. To continue we call WOLFQP changing MAXIT to 100. No other change is necessary.

Inputs:

```
$IN
  EPS = 1.E-10,
  FLAG = .TRUE.,
  IND = 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,1,2,3,4,1,2,
  C = 0.,
  VALUE = 1.,
  N = 26,
  NCON = 12,
  NUNC = 8,
  NINEQ = 4,
  NINV = 3000,
  ITMAX = 100,
  Q = 900 * 0.,
  Q(1,1) = 1.,
  Q(2,2) = 1.,
  Q(3,3) = 1.,
  Q(4,4) = 1.,
  Q(5,5) = 1.,
  Q(6,6) = 1.,
  Q(7,7) = 1.,
  Q(8,8) = 1.,
  Q(9,9) = 1.,
  Q(10,10) = 1.,
  Q(11,11) = 1.,
  Q(12,12) = 1.,
  Q(13,13) = 1.,
  Q(14,14) = 1.,
  Q(15,15) = 1.,
  Q(16,16) = 1.,
  Q(17,17) = 1.,
  Q(18,18) = 1.,
  Q(19,19) = 1.,
  Q(20,20) = 1.,
  A1 = 300*0.,
  A1(1,1) = 1.,
  A1(1,2) = 1.,
  A1(1,3) = 0.,1.,
  A1(1,4) = 0.,1.,
  A1(1,7) = 2*0.,1.,
  A1(1,8) = 2*0.,1.,
  A1(1,9) = 2*0.,1.,
  A1(1,13) = 1.,
  A1(1,14) = 0.,1.,
  A1(1,17) = 3*0.,1.,
  A1(1,18) = 3*0.,1.,
  A1(1,19) = 3*0.,1.,
  A2(1,1) = 300*0.,
  A2(1,1) = 1.,
   A2(1,2) = 1.,
   A2(1,3) = 1.,
   A2(1,4) = 1.,
   A2(1,5) = 1.,
   A2(1,6) = 1.,
      A2(1,7) = 0.,1.,
   A2(1,8) = 0.,1.,
   A2(1,9) = 0.,1.,
   A2(1,10) = 0.,1.,
   A2(1,11) = 0.,1.,
   A2(1,12) = 0.,1.,
   A2(1,13) = 1.,
   A2(1,14) = 1.,
   A2(1,15) = 1.,
   A2(1,16) = 1.,
   A2(1,17) = 0.,1.,
   A2(1,18) = 0.,1.,
   A2(1,19) = 0.,1.,
   A2(1,20) = 0.,1.,
   OBJ = -.8566,-.8187,-.8187,-8187,-.78,-.7701,-.0659,-.0221,-.0130,
      3*0.,-1.,-.9927,-.8047,-.9643,-.0667,-.0158,-.0043,11*0.,
 END
```

```
$OUT
N        = 26.
NCON     = 12.
NUNC     = 8.
NINEQ    = 4.
NEQ      = 2.
NINV     = 3000.
ITMAX    = 100.
ITER     = 16.
IERR     = 0.
C        = 0.0.
VALUE    = -.74708433333331E-02.
$END
          Q
 .1000E+01    0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            .1000E+01     0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            .1000E+01     0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            .1000E+01     0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            .1000E+01     0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            .1000E+01     0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            .1000E+01     0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            .1000E+01     0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            .1000E+01     0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            .1000E+01
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.

0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
 .1000E+01    0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            .1000E+01     0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            .1000E+01     0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            .1000E+01     0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            .1000E+01     0.            0.            0.            0.            0.
0.            0.            0.            0.            0.            .1000E+01     0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            .1000E+01     0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            .1000E+01     0.            0.
0.            0.            0.            0.            0.            0.            0.            0.            .1000E+01     0.
0.            0.            0.            0.            0.            0.            0.            0.            0.            .1000E+01
```

```
      P

-.8566E+00    -.8187E+00    -.8187E+00    -.8187E+00    -.7800E+00    -.7701E+00    -.6590E-01    -.2210E-01    -.1300E-01    0.
0.            0.            -.1000E+01    -.9927E+00    -.8047E+00    -.9643E+00    -.6670E-01    -.1580E-01    -.4300E-02    0.

  A1*X          B1            A1

0.            0.             .1000E+01     .1000E+01    0.            0.            0.            0.            0.            0.
0.            0.            0.            0.             .1000E+01     .1000E+01    0.            0.            0.            0.
0.            0.            0.            0.            0.            0.            0.            0.             .1000E+01     .1000E+01
-.1110E-15    0.            0.            0.            0.            0.            0.            0.            0.            0.

                            0.            0.            0.            0.             .1000E+01    0.            0.            0.
                            0.            0.            0.            0.            0.             .1000E+01    0.            0.
                             .1000E+01    0.            0.            0.            0.            0.            0.            0.
                            0.            0.            0.            0.            0.            0.            0.            0.

                            0.            0.            0.            0.
                            0.            0.            0.            0.
                            0.            0.            0.            0.
                             .1000E+01     .1000E+01     .1000E+01    0.

  A2*X          B2            A2

0.            0.             .1000E+01     .1000E+01     .1000E+01     .1000E+01     .1000E+01     .1000E+01    0.            0.
-.1110E-15    0.            0.            0.            0.            0.            0.            0.             .1000E+01     .1000E+01

                            0.            0.            0.            0.             .1000E+01     .1000E+01     .1000E+01     .1000E+01
                             .1000E+01     .1000E+01     .1000E+01     .1000E+01    0.            0.            0.            0.

                            0.            0.            0.            0.
                             .1000E+01     .1000E+01     .1000E+01     .1000E+01

      X

0.            0.            0.            0.            0.            0.            0.            0.            0.            0.
0.            0.            0.            0.            -.7980E-01     .7980E-01     .3777E-01    -.1313E-01    -.2463E-01    0.
```

After unscrambling the output so that it corresponds to the first problem of this example set one can see the answers are the same.

Example 4

Example 3 illustrated a constrained least-squares problem which consisted of finding a Euclidean projection onto a cone. More general least-squares approximation problems lead, by the same algebraic manipulations which yield the normal equations, to quadratic minimization problems with the Q - matrix not equal to the identity.

Given k values $H_1, \ldots, H_k$, we wish to find n values $x_1, \ldots, x_n$ which minimize $1/2 \overset{k}{\underset{i=1}{}} h_i^2$ where $h_i = \overset{n}{\underset{i=1}{}} C_{ij} x_j - H_i$, the i-th residual, subject to upper and lower bounds on the variables x_j, $j=1,\ldots,n$. In matrix terms we can define a vector residual $h = CX - H$, where C is $k \times n$. Expanding the quadratic gives us the problem:

$$\text{minimize } 1/2\ x^T(C^TC)X \ -(H^TC)x + 1/2\ H^TH$$

$$\text{subject to} \quad \begin{bmatrix} I \\ -I \end{bmatrix} X \leq \begin{bmatrix} u \\ -\ell \end{bmatrix}$$

where u and ℓ are vectors of upper and lower bounds. Here C^TC corresponds to Q, $-H^TC$ to p and $1/2\ H^TH$ to C. (More general linear constraints could also be used.)

The following example, for which we are indebted to George Gray of E22, NSWC, treats the approximation of a given magnetic field. The original data, unscaled, make all the Q-matrix entries very small ($\sim 10^{-12}$). Since values less than EPS1 in magnitude are set to zero in the tableau calculations, we must ensure that EPS1 is small enough to ensure that no significant Q - matrix entries are zeroed. We present three cases, one with EPS1 = 10^{-10} (which bombs), one with EPS1 = 0, and one with EPS1 = 10^{-10} but with the data rescaled (x divided by 10^6, C multiplied by 10^6) so that this difficulty does not arise.

Example 4, Case 1
Inputs:

```
$IN
 EPS = 1.E-10,
 N =  27,
 NCON = 0,
 NUNC = 9,
 NINEQ = 18,
 NINV = 200,
 FLAG = .FALSE.,
 ITMAX = 200,
 VALUE = 1.,
 C = 1065.7777,
 IND = 1,2,3,4,5,6,7,8,9,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
 OBJ = 2.293246044E-5,2.765742302E-6,-2.179388535E-5,-3.785483192E-5,
    -3.739012681E-5,-4.532612771E-5,-4.154215575E-5,-1.86391216E-5,
    1.286575818E-5,3.16621E+6,8.48499E+6,13.06003E+6,1.62557+7,
    1.91037+7,1.77811+7,1.65675+7,12.75075E+6,5.80300E+6,
    -1.05051E+6,-2.82833E+6,-4.35334E+6,-.54186+7,-.63678+7,-.59270+7,
    -.55225+7,-4.23525E+6,-1.93433E+6,
 Q = 8.706679319E-12,3.2939667558E-13,-2.362473067E-12,-1.182179011E-12,
    -4.737248319E-13,-1.97169935E-13,-8.830662389E-14,-4.324526662E-14,-2.291560193E-14,
 Q(1,2) = 3.293967558E-13,7.49676956E-12,3.640488738E-13,-2.343175479E-12,
    -1.17408002E-12,-4.738453526E-13,-1.956924747E-13,-8.802831499E-14,-4.324526662E-14,
 Q(1,3) = -2.362473067E-12,3.640488738E-13,8.384986211E-12,2.517128094E-13,-2.339273365E-12,
 -1.180883408E-12,-4.725728139E-13,-1.956924747E-13,-8.830662389E-14,
 Q(1,4) = -1.182179011E-12,-2.343175479E-12,2.51712809E-13,8.563588812E-12,3.484107527E-13,
   -2.362765444E-12,-1.180883408E-12,-4.738453526E-13,-1.97169935E-13,
 Q(1,5) = -4.737248319E-13,-1.174080021E-12,-2.339273365E-12,3.484107527E-13,7.467745672E-12,
    3.484107527E-13,-2.339273365E-12,-1.174080021E-12,-4.737248319E-13,
 Q(1,6) = -1.97169935E-13,-4.738453526E-13,-1.180883408E-12,-2.362765444E-12,3.484107527E-13,
    8.563588812E-12,2.51712809E-13,-2.34317755479E-12,-1.182179011E-12,
 Q(1,7) = -8.830662389E-14,-1.956924747E-13,-4.725728139E-13,-1.180883408E-12,-2.339273365E-12,
    2.517128094E-13,8.384986211E-12,3.640488738E-13,-2.362473067E-12,
 Q(1,8) = -4.324526662E-14,-8.802831499E-14,-1.956924747E-13,-4.738453526E-13,-1.174080021E-12,
    -2.343175479E-12,3.640488738E-13,7.49676956E-12,3.293967558E-13,
 Q(1,9) = -2.291560193E-14,-4.324526662E-14,-8.830662389E-14,-1.97169935E-13,-4.737248319E-13,
    -1.182179011E-12,-2.362473067E-12,3.293967558E-13,8.706679319E-12,
 A1 = 600*0.,
 A1(1,1) = 1.,
 A1(2,2) = 1.,
 A1(3,3) = 1.,
 A1(4,4) = 1.,
 A1(5,5) = 1.,
 A1(6,6) = 1.,
 A1(7,7) = 1.,
 A1(8,8) = 1.,
 A1(9,9) = 1.,
 A1(10,1) = -1.,
 A1(11,2) = -1.,
 A1(12,3) = -1.,
 A1(13,4) = -1.,
 A1(14,5) = -1.,
 A1(15,6) = -1.,
 A1(16,7) = -1.,
 A1(17,8)= -1.,
 A1(18,9) = -1.,
$END
```

Example 4, Case 1
Outputs:

```
$OUT

N      = 27,

NCON   = 0,

NUNC   = 9,

NINEQ  = 18,

NEQ    = 0,

NINV   = 200,

ITMAX  = 200,

ITER   = 3,

IERR   = 4,

C      = .10657777E+04,

VALUE  = .27208508981644E+03,

$END

       Q

  .8707E-11   .3294E-12  -.2362E-11  -.1182E-11  -.4737E-12  -.1972E-12  -.8831E-13  -.4325E-13  -.2292E-13
  .3294E-12   .7497E-11   .3640E-12  -.2343E-11  -.1174E-11  -.4738E-12  -.1957E-12  -.8803E-13  -.4325E-13
 -.2362E-11   .3640E-12   .8385E-11   .2517E-12  -.2339E-11  -.1181E-11  -.4726E-12  -.1957E-12  -.8831E-13
 -.1182E-11  -.2343E-11   .2517E-12   .8564E-11   .3484E-12  -.2363E-11  -.1181E-11  -.4738E-12  -.1972E-12
 -.4737E-12  -.1174E-11  -.2339E-11   .3484E-12   .7468E-11   .3484E-12  -.2339E-11  -.1174E-11  -.4737E-12
 -.1972E-12  -.4738E-12  -.1181E-11  -.2363E-11   .3484E-12   .8564E-11   .2517E-12  -.2343E-11  -.1182E-11
 -.8831E-13  -.1957E-12  -.4726E-12  -.1181E-11  -.2339E-11   .2517E-12   .8385E-11   .3640E-12  -.2362E-11
 -.4325E-13  -.8803E-13  -.1957E-12  -.4738E-12  -.1174E-11  -.2343E-11   .3640E-12   .7497E-11   .3294E-12
 -.2292E-13  -.4325E-13  -.8831E-13  -.1972E-12  -.4737E-12  -.1182E-11  -.2362E-11   .3294E-12   .8707E-11


       P

  .2293E-04   .2766E-05  -.2179E-04  -.3785E-04  -.3739E-04  -.4533E-04  -.4154E-04  -.1864E-04   .1287E-04

   A1*X         B1              A1

  .1051E+07   .3166E+07   .1000E+01  0.          0.          0.          0.          0.          0.          0.
  .2828E+07   .8485E+07  0.           .1000E+01  0.          0.          0.          0.          0.          0.
  .4353E+07   .1306E+08  0.          0.           .1000E+01  0.          0.          0.          0.          0.
  .5419E+07   .1626E+08  0.          0.          0.           .1000E+01  0.          0.          0.          0.
  .6368E+07   .1910E+08  0.          0.          0.          0.           .1000E+01  0.          0.          0.
  .5927E+07   .1778E+08  0.          0.          0.          0.          0.           .1000E+01  0.          0.
  .5523E+07   .1657E+08  0.          0.          0.          0.          0.          0.           .1000E+01  0.
  .4235E+07   .1275E+08  0.          0.          0.          0.          0.          0.          0.           .1000E+01
  .1934E+07   .5803E+07  0.          0.          0.          0.          0.          0.          0.          0.
 -.1051E+07  -.1051E+07  -.1000E+01  0.          0.          0.          0.          0.          0.          0.
 -.2828E+07  -.2828E+07  0.          -.1000E+01  0.          0.          0.          0.          0.          0.
 -.4353E+07  -.4353E+07  0.          0.          -.1000E+01  0.          0.          0.          0.          0.
 -.5419E+07  -.5419E+07  0.          0.          0.          -.1000E+01  0.          0.          0.          0.
 -.6368E+07  -.6368E+07  0.          0.          0.          0.          -.1000E+01  0.          0.          0.
 -.5927E+07  -.5927E+07  0.          0.          0.          0.          0.          -.1000E+01  0.          0.
 -.5523E+07  -.5523E+07  0.          0.          0.          0.          0.          0.          -.1000E+01  0.
 -.4235E+07  -.4235E+07  0.          0.          0.          0.          0.          0.          0.          -.1000E+01
 -.1934E+07  -.1934E+07  0.          0.          0.          0.          0.          0.          0.          0.
```

Example 4, Case 1
Outputs Cont.:

```
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                            .1000E+01
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           -.1000E+01

       X

.1051E+07     .2828E+07     .4353E+07     .5419E+07     .6368E+07     .5927E+07     .5523E+07     .4235E+07     .1934E+07
```

Example 4, Case 2
Inputs:

```
$IN
 EPS = 0.,
 N =  27,
 NCON = 0,
 NUNC = 9,
 NINEQ = 18,
 NINV = 200,
 FLAG = .FALSE.,
 ITMAX = 200,
 VALUE = 1.,
 C = 1065.7777,
 IND = 1,2,3,4,5,6,7,8,9,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
 OBJ = 2.293246044E-5,2.765742302E-6,-2.179388535E-5,-3.785483192E-5,
    -3.739012681E-5,-4.532612771E-5,-4.154215575E-5,-1.86391216E-5,
    1.286575818E-5,3.16621E+6,8.48499E+6,13.06003E+6,1.62557+7,
    1.91037+7,1.77811+7,1.65675+7,12.75075E+6,5.80300E+6,
    -1.05051E+6,-2.82833E+6,-4.35334E+6,-.54186+7,-.63678+7,-.59270+7,
    -.55225+7,-4.23525E+6,-1.93433E+6,
 Q = 8.706679319E-12,3.2939667558E-13,-2.362473067E-12,-1.182179011E-12,
    -4.737248319E-13,-1.97169935E-13,-8.830662389E-14,-4.324526662E-14,-2.291560193E-14,
 Q(1,2) = 3.293967558E-13,7.49676956E-12,3.640488738E-13,-2.343175479E-12,
     -1.17408002E-12,-4.738453526E-13,-1.956924747E-13,-8.802831499E-14,-4.324526662E-14,
 Q(1,3) = -2.362473067E-12,3.640488738E-13,8.384986211E-12,2.517128094E-13,-2.339273365E-12,
 -1.180883408E-12,-4.725728139E-13,-1.956924747E-13,-8.830662389E-14,
 Q(1,4) = -1.182179011E-12,-2.343175479E-12,2.51712809E-13,8.563588812E-12,3.484107527E-13,
   -2.362765444E-12,-1.180883408E-12,-4.738453526E-13,-1.97169935E-13,
 Q(1,5) = -4.737248319E-13,-1.174080021E-12,-2.339273365E-12,3.484107527E-13,7.467745672E-12,
    3.484107527E-13,-2.339273365E-12,-1.174080021E-12,-4.737248319E-13,
 Q(1,6) = -1.97169935E-13,-4.738453526E-13,-1.180883408E-12,-2.362765444E-12,3.484107527E-13,
    8.563588812E-12,2.51712809E-13,-2.3431755479E-12,-1.182179011E-12,
 Q(1,7) = -8.830662389E-14,-1.956924747E-13,-4.725728139E-13,-1.180883408E-12,-2.339273365E-12,
    2.517128094E-13,8.384986211E-12,3.640488738E-13,-2.362473067E-12,
 Q(1,8) = -4.324526662E-14,-8.802831499E-14,-1.956924747E-13,-4.738453526E-13,-1.174080021E-12,
    -2.343175479E-12,3.640488738E-13,7.49676956E-12,3.293967558E-13,
 Q(1,9) = -2.291560193E-14,-4.324526662E-14,-8.830662389E-14,-1.97169935E-13,-4.737248319E-13,
    -1.182179011E-12,-2.362473067E-12,3.293967558E-13,8.706679319E-12,
 A1 = 600*0.,
 A1(1,1) = 1.,
 A1(2,2) = 1.,
 A1(3,3) = 1.,
 A1(4,4) = 1.,
 A1(5,5) = 1.,
 A1(6,6) = 1.,
 A1(7,7) = 1.,
 A1(8,8) = 1.,
 A1(9,9) = 1.,
 A1(10,1) = -1.,
 A1(11,2) = -1.,
 A1(12,3) = -1.,
 A1(13,4) = -1.,
 A1(14,5) = -1.,
 A1(15,6) = -1.,
 A1(16,7) = -1.,
 A1(17,8)= -1.,
 A1(18,9) = -1.,
$END
```

Example 4, Case 2

Outputs:

```
$SOUT

N       = 27,

NCON    = 0,

NUNC    = 9,

NINEQ   = 18,

NEQ     = 0,

NINV    = 200,

ITMAX   = 200,

ITER    = 12,

IERR    = 0,

C       = .10657777E+04,

VALUE   = .7597274307163E+01,

$END

      Q

 .8707E-11   .3294E-12  -.2362E-11  -.1182E-11  -.4737E-12  -.1972E-12  -.8831E-13  -.4325E-13  -.2292E-13
 .3294E-12   .7497E-11   .3640E-12  -.2343E-11  -.1174E-11  -.4738E-12  -.1957E-12  -.8803E-13  -.4325E-13
-.2362E-11   .3640E-12   .8385E-11   .2517E-12  -.2339E-11  -.1181E-11  -.4726E-12  -.1957E-12  -.8831E-13
-.1182E-11  -.2343E-11   .2517E-12   .8564E-11   .3484E-12  -.2363E-11  -.1181E-11  -.4738E-12  -.1972E-12
-.4737E-12  -.1174E-11  -.2339E-11   .3484E-12   .7468E-11   .3484E-12  -.2339E-11  -.1174E-11  -.4737E-12
-.1972E-12  -.4738E-12  -.1181E-11  -.2363E-11   .3484E-12   .8564E-11   .2517E-12  -.2343E-11  -.1182E-11
-.8831E-13  -.1957E-12  -.4726E-12  -.1181E-11  -.2339E-11   .2517E-12   .8385E-11   .3640E-12  -.2362E-11
-.4325E-13  -.8803E-13  -.1957E-12  -.4738E-12  -.1174E-11  -.2343E-11   .3640E-12   .7497E-11   .3294E-12
-.2292E-13  -.4325E-13  -.8831E-13  -.1972E-12  -.4737E-12  -.1182E-11  -.2362E-11   .3294E-12   .8707E-11


      P

 .2293E-04   .2766E-05  -.2179E-04  -.3785E-04  -.3739E-04  -.4533E-04  -.4154E-04  -.1864E-04   .1287E-04

  A1*X         B1                A1

 .2112E+07   .3166E+07   .1000E+01  0.          0.          0.          0.          0.          0.          0.          0.
 .5657E+07   .8485E+07  0.           .1000E+01  0.          0.          0.          0.          0.          0.          0.
 .8707E+07   .1306E+08  0.          0.           .1000E+01  0.          0.          0.          0.          0.          0.
 .1084E+08   .1626E+08  0.          0.          0.           .1000E+01  0.          0.          0.          0.          0.
 .1274E+08   .1910E+08  0.          0.          0.          0.           .1000E+01  0.          0.          0.          0.
 .1185E+08   .1778E+08  0.          0.          0.          0.          0.           .1000E+01  0.          0.          0.
 .1105E+08   .1657E+08  0.          0.          0.          0.          0.          0.           .1000E+01  0.          0.
 .8470E+07   .1275E+08  0.          0.          0.          0.          0.          0.          0.           .1000E+01  0.
 .3869E+07   .5803E+07  0.          0.          0.          0.          0.          0.          0.          0.           .1000E+01
-.2112E+07  -.1051E+07  -.1000E+01  0.          0.          0.          0.          0.          0.          0.          0.
-.5657E+07  -.2828E+07  0.          -.1000E+01  0.          0.          0.          0.          0.          0.          0.
-.8707E+07  -.4353E+07  0.          0.          -.1000E+01  0.          0.          0.          0.          0.          0.
-.1084E+08  -.5419E+07  0.          0.          0.          -.1000E+01  0.          0.          0.          0.          0.
-.1274E+08  -.6368E+07  0.          0.          0.          0.          -.1000E+01  0.          0.          0.          0.
-.1185E+08  -.5927E+07  0.          0.          0.          0.          0.          -.1000E+01  0.          0.          0.
-.1105E+08  -.5523E+07  0.          0.          0.          0.          0.          0.          -.1000E+01  0.          0.
-.8470E+07  -.4235E+07  0.          0.          0.          0.          0.          0.          0.          -.1000E+01  0.
-.3869E+07  -.1934E+07  0.          0.          0.          0.          0.          0.          0.          0.          -.1000E+01
```

Example 4, Case 2
Outputs Cont.:

```
                     0.
                     0.
                     0.
                     0.
                     0.
                     0.
                     0.
                     0.
                      .1000E+01
                     0.
                     0.
                     0.
                     0.
                     0.
                     0.
                     0.
                     0.
                     -.1000E+01

     X
 .2112E+07  .5657E+07  .8707E+07  .1084E+08  .1274E+08  .1185E+08  .1105E+08  .8470E+07  .3869E+07
```

Example 4, Case 3
Inputs:

```
$IN
 EPS = 1.E-10,
 N =  27,
 NCON = 0,
 NUNC = 9,
 NINEQ = 18,
 NINV = 200,
 FLAG = .FALSE.,
 ITMAX = 200,
 VALUE = 1.,
 C = 1065.7777,
 IND = 1,2,3,4,5,6,7,8,9,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,
 OBJ = 2.293246044E+1,2.765742302F+0,-2.179388535E+1,-3.785483192E+1,
     -3.739012681E+1,-4.532612771E+1,-4.154215575E+1,-1.863912161E+1,
     1.286575818E+1,3.16621E+0,8.48499E+0,13.06003E+0,1.62557E+1,
     1.91037E+1,1.77811E+1,1.65675E+1,12.75075E+0,5.80300E+0,
     -1.05051E+0,-2.82833E+0,-4.35334E+0,-.54186E+1,-.63678E+1,-.59270E+1,
     -.55225E+1,-4.23525E+0,-1.93433E+0,
 Q = 8.706679319E+0,3.293967558E-1,-2.362473067E+0,-1.182179011E+0,
     -4.737248319E-1,-1.97169935E-1,-8.830662389E-2,-4.324526662E-2,-2.291560193E-2,
 Q(1,2) = 3.293967558E-1,7.49676956E+0,3.640488738E-1,-2.343175479E+0,
      -1.17408002E+0,-4.738453526E-1,-1.956924747E-1,-8.802831499E-2,-4.324526662E-2,
 Q(1,3) = -2.362473067E+0,3.640488738E-1,8.384986211E+0,2.517128094E-1,-2.339273365E+0,
 -1.180883408E+0,-4.725728139E-1,-1.956924747E-1,-8.830662389E-2,
 Q(1,4) = -1.182179011E+0,-2.343175479E+0,2.51712809E-1,8.563588812E+0,3.484107527E-1,
    -2.362765444E+0,-1.180883408E+0,-4.738453526E-1,-1.97169935E-1,
 Q(1,5) = -4.737248319E-1,-1.174080021E+0,-2.339273365E+0,3.484107527E-1,7.467745672E+0,
     3.484107527E-1,-2.339273365E+0,-1.174080021E+0,-4.737248319E-1,
 Q(1,6) = -1.97169935E-1,-4.738453526E-1,-1.180883408E+0,-2.362765444E+0,3.484107527E-1,
     8.563588812E+0,2.51712809E-1,-2.3431755479E+0,-1.182179011E+0,
 Q(1,7) = -8.830662389E-2,-1.956924747E-1,-4.725728139E-1,-1.180883408E+0,-2.339273365E+0,
     2.517128094E-1,8.384986211E+0,3.640488738E-1,-2.362473067E+0,
 Q(1,8) = -4.324526662E-2,-8.802831499E-2,-1.956924747E-1,-4.738453526E-1,-1.174080021E+0,
     -2.343175479E+0,3.640488738E-1,7.49676956E+0,3.293967558E-1,
 Q(1,9) = -2.291560193E-2,-4.324526662E-2,-8.830662389E-2,-1.97169935E-1,-4.737248319E-1,
     -1.182179011E+0,-2.362473067E+0,3.293967558E-1,8.706679319E+0,
 A1 = 600*0.,
 A1(1,1) = 1.,
 A1(2,2) = 1.,
 A1(3,3) = 1.,
 A1(4,4) = 1.,
 A1(5,5) = 1.,
 A1(6,6) = 1.,
 A1(7,7) = 1.,
 A1(8,8) = 1.,
 A1(9,9) = 1.,
 A1(10,1) = -1.,
 A1(11,2) = -1.,
 A1(12,3) = -1.,
 A1(13,4) = -1.,
 A1(14,5) = -1.,
 A1(15,6) = -1.,
 A1(16,7) = -1.,
 A1(17,8)= -1.,
 A1(18,9) = -1.,
$END
```

Example 4, Case 3
Outputs:

```
$OUT

N        = 27,

NCON     = 0,

NUNC     = 9,

NINEQ    = 18,

NEQ      = 0,

NINV     = 200,

ITMAX    = 200,

ITER     = 9,

IERR     = 0,

C        = .10657777E+04,

VALUE    = .75972743071738E+01,

$END

          Q

  .8707E+01   .3294E+00  -.2362E+01  -.1182E+01  -.4737E+00  -.1972E+00  -.8831E-01  -.4325E-01  -.2292E-01
  .3294E+00   .7497E+01   .3640E+00  -.2343E+01  -.1174E+01  -.4738E+00  -.1957E+00  -.8803E-01  -.4325E-01
 -.2362E+01   .3640E+00   .8385E+01   .2517E+00  -.2339E+01  -.1181E+01  -.4726E+00  -.1957E+00  -.8831E-01
 -.1182E+01  -.2343E+01   .2517E+00   .8564E+01   .3484E+00  -.2363E+01  -.1181E+01  -.4738E+00  -.1972E+00
 -.4737E+00  -.1174E+01  -.2339E+01   .3484E+00   .7468E+01   .3484E+00  -.2339E+01  -.1174E+01  -.4737E+00
 -.1972E+00  -.4738E+00  -.1181E+01  -.2363E+01   .3484E+00   .8564E+01   .2517E+00  -.2343E+01  -.1182E+01
 -.8831E-01  -.1957E+00  -.4726E+00  -.1181E+01  -.2339E+01   .2517E+00   .8385E+01   .3640E+00  -.2362E+01
 -.4325E-01  -.8803E-01  -.1957E+00  -.4738E+00  -.1174E+01  -.2343E+01   .3640E+00   .7497E+01   .3294E+00
 -.2292E-01  -.4325E-01  -.8831E-01  -.1972E+00  -.4737E+00  -.1182E+01  -.2362E+01   .3294E+00   .8707E+01


          P

  .2293E+02   .2766E+01  -.2179E+02  -.3785E+02  -.3739E+02  -.4533E+02  -.4154E+02  -.1864E+02   .1287E+02

   A1*X        B1              A1

  .2112E+01   .3166E+01   .1000E+01  0.          0.          0.          0.          0.          0.          0.          0.
  .5657E+01   .8485E+01  0.           .1000E+01  0.          0.          0.          0.          0.          0.          0.
  .8707E+01   .1306E+02  0.          0.           .1000E+01  0.          0.          0.          0.          0.          0.
  .1084E+02   .1626E+02  0.          0.          0.           .1000E+01  0.          0.          0.          0.          0.
  .1274E+02   .1910E+02  0.          0.          0.          0.           .1000E+01  0.          0.          0.          0.
  .1185E+02   .1778E+02  0.          0.          0.          0.          0.           .1000E+01  0.          0.          0.
  .1105E+02   .1657E+02  0.          0.          0.          0.          0.          0.           .1000E+01  0.          0.
  .8470E+01   .1275E+02  0.          0.          0.          0.          0.          0.          0.           .1000E+01  0.
  .3869E+01   .5803E+01  0.          0.          0.          0.          0.          0.          0.          0.           .1000E+01
 -.2112E+01  -.1051E+01  -.1000E+01  0.          0.          0.          0.          0.          0.          0.          0.
 -.5657E+01  -.2828E+01  0.          -.1000E+01  0.          0.          0.          0.          0.          0.          0.
 -.8707E+01  -.4353E+01  0.          0.          -.1000E+01  0.          0.          0.          0.          0.          0.
 -.1084E+02  -.5419E+01  0.          0.          0.          -.1000E+01  0.          0.          0.          0.          0.
 -.1274E+02  -.6368E+01  0.          0.          0.          0.          -.1000E+01  0.          0.          0.          0.
 -.1185E+02  -.5927E+01  0.          0.          0.          0.          0.          -.1000E+01  0.          0.          0.
 -.1105E+02  -.5523E+01  0.          0.          0.          0.          0.          0.          -.1000E+01  0.          0.
 -.8470E+01  -.4235E+01  0.          0.          0.          0.          0.          0.          0.          -.1000E+01  0.
 -.3869E+01  -.1934E+01  0.          0.          0.          0.          0.          0.          0.          0.          -.1000E+01
```

Example 4, Case 3
Outputs Cont.:

```
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                            .1000E+01
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           0.
                           -.1000E+01

       X
.2112E+01     .5657E+01     .8707E+01     .1084E+02     .1274E+02     .1185E+02     .1105E+02     .8470E+01     .3869E+01
```

REFERENCES

1. J. W. Wingate, LINOPT, A FORTRAN Routine for Solving Linear Programming Problems, NSWC TR 80-413

2. P. Wolfe, The Simplex Method for Quadratic Programming, Econometrica v.27 pp. 382-298 (1959)

3. G. Zoutendijk, Methods of Feasible Directions, Elsevier, Amsterdam, 1960

APPENDIX A

LISTING

```
      13/74    OPT=1                         FTN 4.6+452        82/01/28. 14.34.48        PAGE    1

      SUBROUTINE WOLFUP(Q,A1,A2,OBJ,IND,IPASS,C,VALUE,SCR,Y,FLAG,EPS)  WOLFUP     2
C                                                                      LINE       2
C--------------------------------------------------------------------- LINE       3
C                                                                      LINE       4
C     QUADRATIC PROGRAMMING BY WOLFE"S METHOD                          WOLFUP     4
C                                                                      WOLFUP     5
C          DETERMINE A VECTOR OF UNKNOWNS X WHICH                      WOLFUP     6
C                                                                      WOLFUP     7
C                                                                      WOLFUP     8
C                            T              T                          WOLFUP     9
C        MINIMIZE  1/2. * X * Q * X  +  P * X + C                      WOLFUP    10
C                                                                      WOLFUP    11
C                                                                      WOLFUP    12
C          SUCH THAT                                                   WOLFUP    13
C                                                                      WOLFUP    14
C               A1 * X <=  B1                                          WOLFUP    15
C                                                                      WOLFUP    16
C               A2 * X = B2                                            WOLFUP    17
C                                                                      WOLFUP    18
C                                                                      WOLFUP    19
C        ELEMENTS OF THE VECTOR X CAN HAVE NON-NEGATIVITY CONSTRAINTS  WOLFUP    20
C        IMPLICITLY HANDLED USING IND, NCON, AND NUNC(SEE TEXT         WOLFUP    21
C        FOLLOWING).  THUS,                                            WOLFUP    22
C                                                                      WOLFUP    23
C          X(IND(I)) >= 0.      I = 1,...,NCON                         WOLFUP    24
C                                                                      WOLFUP    25
C          X(IND(I)) UNCONSTRAINED IN SIGN    I = NCON+1,...,NCON+NUNC WOLFUP    26
C                                                                      WOLFUP    27
C          WHERE NCON IS THE NUMBER OF SIGN CONSTRAINED VARIABLES AND  WOLFUP    28
C          NUNC IS THE NUMBER OF FREE VARIABLES.                       WOLFUP    29
C                                                                      WOLFUP    30
C                                                                      WOLFUP    31
C                         ------------                                 WOLFUP    32
C                                                                      WOLFUP    33
C                                                                      WOLFUP    34
C     INPUTS--THE FOLLOWING VARIABLES AND ARRAYS MUST BE DEFINED ON    WOLFUP    35
C             ENTRY                                                    WOLFUP    36
C                                                                      WOLFUP    37
C       IPASS        INTEGER ARRAY CONTAINING SCALAR VARIABLES DEFINING WOLFUP   38
C                    THE PROBLEM.                                      WOLFUP    39
C                                                                      WOLFUP    40
C       IPASS(1)     N  THE DIMENSION OF THE PROBLEM (DIMENSION(X) +   WOLFUP    41
C                        + DIMENSION(B1) + DIMENSION(B2)).             WOLFUP    42
C                                                                      WOLFUP    43
C       IPASS(2)     ITMAX  THE MAXIMUM NUMBER OF ITERATIONS DESIRED.  WOLFUP    44
C                                                                      WOLFUP    45
C       IPASS(5)     NCON   THE NUMBER OF SIGN CONSTRAINED VARIABLES.  WOLFUP    46
C                                                                      WOLFUP    47
C       IPASS(6)     NUNC   THE NUMBER OF UNCONSTRAINED IN SIGN VARIABLE WOLFUP  48
C                                                                      WOLFUP    49
C       IPASS(7)     NINEQ   THE NUMBER OF INEQUALITY CONSTRAINTS.     WOLFUP    50
C                                                                      WOLFUP    51
C       IPASS(8)     NINVT  THE NUMBER OF ITERATIONS BETWEEN           WOLFUP    52
C                    INVERSIONS TO CONTROL ROUND-OFF.                  WOLFUP    53
C                                                                      WOLFUP    54
C       IPASS(9)     NQD   DECLARED ROW DIMENSION OF Q.                WOLFUP    55
C                                                                      WOLFUP    56
C       IPASS(10)    N1D   DECLARED ROW DIMENSION OF A1.
```

```
C                                                                   WOLFQP     57
C       IPASS(11)   NRD   DECLARED ROW DIMENSION OF A2.             WOLFQP     58
C                                                                   WOLFQP     59
C       Q           REAL ARRAY CONTAINING A POSITIVE SEMI-DEFINITE  WOLFQP     60
C                   MATRIX CORRESPONDING TO THE QUADRATIC           WOLFQP     61
C                   PORTION OF THE OBJECTIVE FUNCTION.              WOLFQP     62
C                                                                   WOLFQP     63
C       A1          REAL ARRAY CONTAINING THE INEQUALITY CONSTRAINTS.  WOLFQP  64
C                                                                   WOLFQP     65
C       A2          REAL ARRAY CONTAINING THE EQUALITY CONTISTRAINTS.  WOLFQP  66
C                                                                   WOLFQP     67
C       OBJ         REAL ARRAY CONTAINING THE OBJECTIVE             WOLFQP     68
C                   FUNCTION OF THE CORRESPONDING LINEAR            WOLFQP     69
C                   PROGRAMMING PROBLEM.                            WOLFQP     70
C                            T    T    T                            WOLFQP     71
C                   OBJ = [P ,B1 ,B2 ]                              WOLFQP     72
C                                                                   WOLFQP     73
C       IND         INTEGER ARRAY USED TO ORDER X INTO SIGN         WOLFQP     74
C                   CONSTRAINED AND SIGN UNCONSTRAINED PARTITIONS.  WOLFQP     75
C                   ALSO USED TO INDEX WHICH ROWS OF THE CONSTRAINT WOLFQP     76
C                   MATRICES [A1, A2] ARE TO BE USED (IF ANY).      WOLFQP     77
C                   THUS                                            WOLFQP     78
C                                                                   WOLFQP     79
C                      X(IND(I)) >= 0.        I = 1,... NCON        WOLFQP     80
C                      X(IND(I))    FREE      I = NCON+1,... NCON+NUNC  WOLFQP 81
C                      SUM OVER I     A1(IND(J+K),I)*X(I) <= B1(J)  WOLFQP     82
C                         K = NCON + NUNC;      J = 1, ...NINEQ     WOLFQP     83
C                      SUM OVER I     A2(IND(J+K),I)*X(I) = B2(J)   WOLFQP     84
C                         K = NCON + NUNC + NINEQ;    J = 1,...N-K  WOLFQP     85
C                                                                   WOLFQP     86
C       C           REAL SCALAR CONTAINING THE CONSTANT TERM        WOLFQP     87
C                   OF THE QUADRATIC PROBLEM.                       WOLFQP     88
C                                                                   WOLFQP     89
C       VALUE       ON INPUT IF QUADRATIC OBJECTIVE FUNCTION        WOLFQP     90
C                   IS TO BE EVALUATED VALUE SHOULD BE SET NOT EQUAL  WOLFQP   91
C                   TO ZERO.  IF VALUE IS ZERO THE                  WOLFQP     92
C                   QUADRATIC OBJECTIVE FUNCTION IS                 WOLFQP     93
C                   NOT EVALUATED.                                  WOLFQP     94
C                                                                   WOLFQP     95
C       FLAG        LOGICAL VARIABLE.  ON INITIAL CALL TO WOLFQP THIS  WOLFQP  96
C                   VARIABLE MUST BE FALSE.  IT IS USED IF WOLFQP IS  WOLFQP   97
C                   RECALLED AND THE TABLEAU IS NOT TO BE RESET.  IF  WOLFQP   98
C                   THE ITERATION LIMIT HAS BEEN REACHED PREVIOUSLY WOLFQP     99
C                   WOLFQP CAN BE RECALLED WITH THE FLAG SET TO TRUE.  WOLFQP 100
C                   NO OTHER CHANGE IS NECESSARY OR ADVISED.        WOLFQP    101
C                   COMPUTATION BEGINS WHERE PREVIOUS ITERATIONS    WOLFQP    102
C                   STOPPED. THE ITERATION COUNTER IS RESET TO ZERO.  WOLFQP  103
C                                                                   WOLFQP    104
C       EPSI        REAL SCALAR. THIS VARIABLE REPRESENTS ZERO      WOLFQP    105
C                   TOLERANCE.  CONSTRAINT VIOALTIONS ON TABLEAU    WOLFQP    106
C                   ENTRIES LESS THAN OR EQUAL TO EPS ARE IGNORED.  WOLFQP    107
C                                                                   WOLFQP    108
C                                                                   WOLFQP    109
C     OUTPUTS--THE FOLLOWING VARIABLES AND ARRAYS ARE DEFINED       WOLFQP    110
C              OR REDEFINED ON EXIT.                                WOLFQP    111
C                                                                   WOLFQP    112
C       IPASS(3)    ITER NUMBER OF ITERATIONS COMPLETED DURING      WOLFQP    113
```

```
C                 THE CURRENT CALL TO WOLFQP.                           WOLFQP   114
C                                                                       WOLFQP   115
C      IPASS(4)   IERR   ERROR INDICATOR                                WOLFQP   116
C                     0  --OPTIMUM FOUND                                WOLFQP   117
C                     1  --INCONSISTENT CONSTRAINTS                     WOLFQP   118
C                     2  --ITERATION LIMIT REACHED                      WOLFQP   119
C                     3  --INVERSION FAILED(BAD INITIAL BASIS)          WOLFQP   120
C                   4+L--FAILED TO SOLVE QUADRATIC PROBLEM              WOLFQP   121
C                          REASON FOR FAILURE IS DUE TO ERROR L         WOLFQP   122
C                                                                       WOLFQP   123
C                                                                       WOLFQP   124
C      VALUE      IF ON INPUT VALUE WAS NOT ZERO VALUE RETURNS          WOLFQP   125
C                 THE VALUE OF THE QUADRATIC OBJECTIVE FUNCTION.        WOLFQP   126
C                                                                       WOLFQP   127
C      Y          REAL ARRAY CONTAINING THE SOLUTION X.                 WOLFQP   128
C                 THE SOLUTION HAS BEEN                                 WOLFQP   129
C                 UNSCRAMBLED BY THE IND ARRAY AND THUS                 WOLFQP   130
C                 CORRESPONDS TO THE ORIGINAL ORDERING.                 WOLFQP   131
C                                                                       WOLFQP   132
C      SCR        PORTIONS OF SCR HAVE BEEN USED                        WOLFQP   133
C                 BY THE PROGRAM.  THESE WILL NOT BE OF                 WOLFQP   134
C                 GENERAL USE BUT THEIR DEFINITIONS.                    WOLFQP   135
C                 ARE GIVEN BELOW.                                      WOLFQP   136
C                                                                       WOLFQP   137
C      SCR(1 TO 2*N+1)                  BL   ARRAY OF LOWER BOUNDS      WOLFQP   138
C      SCR(2*N+2 TO 4*N+2)              BU   ARRAY OF UPPER BOUNDS      WOLFQP   139
C      SCR(4*N+3 TO 6*N+3)              X    PRIMAL SOLUTION ARRAY      WOLFQP   140
C      SCR(6*N+4 TO 8*N+4)              U    DUAL SOLUTION ARRAY        WOLFQP   141
C      SCR(8*N+5 TO 10*N+5)             BASIC FLAGS USED INTERNALLY     WOLFQP   142
C      SCR(10*N+6 TO 11*N+5)            AROW  SCRATCH SPACE             WOLFQP   143
C      SCR(11*N+6 TO N**2+11*N+5)       L    THE INVERSE MATRIX         WOLFQP   144
C      SCR(N**2+11*N+6 TO N**2+13*N+6)  K    BASIC SOLUTION KEY         WOLFQP   145
C                                                                       WOLFQP   146
C                                                                       WOLFQP   147
C      MINIMUM DECLARED ARRAY SIZES                                     WOLFQP   148
C                                                                       WOLFQP   149
C         Q(NCON+NUNC,NCON+NUNC)                                        WOLFQP   150
C         A1(NINEQ,NCON+NUNC)                                           WOLFQP   151
C         A2(N-(NCON+NUNC+NINEQ),NCON+NUNC)                             WOLFQP   152
C         OBJ(N)                                                        WOLFQP   153
C         SCR(N**2+13*N+6)                                              WOLFQP   154
C         Y(NCON+NUNC)                                                  WOLFQP   155
C         IND(N)                                                        WOLFQP   156
C         IPASS(11)                                                     WOLFQP   157
C                                                                       WOLFQP   158
C                                                                       WOLFQP   159
C                                                                       WOLFQP   160
C      THE FOLLOWING ARE FURTHER DESCRIPTIONS OF INTERNAL VARIABLES.    WOLFQP   161
C                                                                       WOLFQP   162
C                                                                       WOLFQP   163
C      BL         ARRAY OF LOWER BOUNDS.                         (REAL) WOLFQP   164
C      BU         ARRAY OF UPPER BOUNDS.                         (REAL) WOLFQP   165
C                                                                       WOLFQP   166
C      K          BASIC SOLUTION KEY.                         (INTEGER) WOLFQP   167
C                 K, IN CONJUNCTION WITH X, SPECIFIES A                 WOLFQP   168
C                 PARTICULAR BASIC SOLUTION.  THE EQUATIONS             WOLFQP   169
C                 RELATING  X(N+I), I = 1,....,N  TO                    WOLFQP   170
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      C                     X(J), J = 1,...,N  (THE CONSTRAINT EQUATIONS)         WOLFQP    171
      C                     CAN BE SOLVED FOR VARIOUS COMBINATIONS OF             WOLFQP    172
      C                     M  VARIABLES (SECONDARY VARIABLES) IN TERMS           WOLFQP    173
      C                     OF THE REMAINING  N  VARIABLES (PRIMARY               WOLFQP    174
      C                     VARIABLES).  K, A PERMUTATION OF  (1,...,N+M),        WOLFQP    175
      C                     SPECIFIES SUCH A PARTITION INTO PRIMARY AND           WOLFQP    176
      C                     SECONDARY VARIABLES.  K(1),...,K(N)  ARE THE          WOLFQP    177
      C                     INDICES OF THE PRIMARY VARIABLES.  K(N+1),...,        WOLFQP    178
      C                     K(N+M)  ARE THE INDICES OF THE SECONDARY              WOLFQP    179
      C                     VARIABLES.  FOR THE DUAL VARIABLES U(J),              WOLFQP    180
      C                     J = 1,...,N+M, PRIMARY AND SECONDARY INDICES          WOLFQP    181
      C                     SWITCH ROLES.  U(K(N+1)),...,U(K(N+M)) BEING          WOLFQP    182
      C                     PRIMARY.  A BASIC SOLUTION IS SPECIFIED BY            WOLFQP    183
      C                     SETTING EACH PRIMAL PRIMARY VARIABLE TO               WOLFQP    184
      C                     EITHER OF ITS BOUNDS AND EACH DUAL PRIMARY            WOLFQP    185
      C                     VARIABLE TO ZERO.  THE INPUT VALUES OF THE            WOLFQP    186
      C                     PRIMAL PRIMARY VARIABLES ARE SWITCHED TO THE          WOLFQP    187
      C                     OPPOSITE BOUND IF NECESSARY IN ORDER TO CREATE        WOLFQP    188
      C                     A DUAL-FEASIBLE BASIC SOLUTION.                       WOLFQP    189
      C                                                                           WOLFQP    190
      C          X          PRIMAL SOLUTION ARRAY.                         (REAL) WOLFQP    191
      C                     X(K(J))  MUST BE SET TO EITHER  BL(K(J)) OR           WOLFQP    192
      C                     BU(K(J)), J = 1,...,N. THESE ARE DEFAULT              WOLFQP    193
      C                     VALUES TO BE USED WHEN A VANISHING  U(K(J))           WOLFQP    194
      C                     MAKES  X(K(J))  INDETERMINATE IN SETTING UP           WOLFQP    195
      C                     A DUAL-FEASIBLE SOLUTION.                             WOLFQP    196
      C                                                                           WOLFQP    197
      C          K          BASIC SOLUTION KEY.                         (INTEGER) WOLFQP    198
      C                     SET FOR THE CURRENT BASIS.                            WOLFQP    199
      C                                                                           WOLFQP    200
      C          X          PRIMAL SOLUTION ARRAY.                         (REAL) WOLFQP    201
      C                                                                           WOLFQP    202
      C          U          DUAL SOLUTION ARRAY.                           (REAL) WOLFQP    203
      C                     U(J)  IS THE DUAL VARIABLE (LAGRANGE                  WOLFQP    204
      C                     MULTIPLIER) FOR THE CONSTRAINTS ON  X(J).             WOLFQP    205
      C                     IT IS POSITIVE IF THE UPPER BOUND IS ACTIVE,          WOLFQP    206
      C                     NEGATIVE IF THE LOWER BOUND IS ACTIVE.                WOLFQP    207
      C                                                                           WOLFQP    208
      C          E          INVERSE MATRIX ARRAY.                          (REAL) WOLFQP    209
      C                     X(I)  =  SUM (J = 1,...,N)  E(I,J) * X(K(J)),         WOLFQP    210
      C                                                         I = 1,...,N.      WOLFQP    211
      C                                                                           WOLFQP    212
      C                           ------------                                    WOLFQP    213
      C                                                                           WOLFQP    214
      C                                                                           LINE        2
      C-------------------------------------------------------------------------- LINE        3
      C                                                                           LINE        4
            DIMENSION U(1), OBJ(1), IPASS(1), SCR(1), Y(1), IND(1)                WOLFQP    216
            LOGICAL FLAG                                                          WOLFQP    217
      C                                                                           WOLFQP    218
            COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                     /XXXQP/     2
            COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21           /XXXQP/     3
            COMMON /XXXQP/ BIGM, NINVT, PHASE1                                    /XXXQP/     4
            LOGICAL NEGV, PHASE1                                                  /XXXQP/     5
      C                                                                           LINE        2
      C-------------------------------------------------------------------------- LINE        3
      C                                                                           LINE        4
      SUBROUTINE WOLFQP     73/74    OPT=1                    FTN 4.6+452        82/01/28. 14.34.48       PAGE     5

C     THE VARIABLE  BIGM  REPRESENTS A VERY LARGE NUMBER.  THE DEFAULT   WOLFQP    221
C     VALUE IS 1.E100.  THE USER MAY RESET THIS VALUE IF SO DESIRED.     WOLFQP    222
C     BIGM OR -BIGM MAY BE USED TO FILL IN MISSING UPPER OR LOWER        WOLFQP    223
C     BOUNDS.  THIS CONSTANT IS MACHINE DEPENDENT.                       WOLFQP    224
C                                                                        WOLFQP    225
      DATA BIGM /1.E+100/                                                WOLFQP    226
C                                                                        LINE        2
C----------------------------------------------------------------------- LINE        3
C                                                                        LINE        4
      IF(FLAG) GO TO 105                                                 WOLFQP    228
      EPS = EPS1                                                         WOLFQP    229
      N = IPASS(1)                                                       WOLFQP    230
      N1 = IPASS(5) + IPASS(6)                                           WOLFQP    231
      N11 = N1 + 1                                                       WOLFQP    232
      N2 = IPASS(7) + N1                                                 WOLFQP    233
      N21 = N2 + 1                                                       WOLFQP    234
      NQD = IPASS(9)                                                     WOLFQP    235
      N1D = IPASS(10)                                                    WOLFQP    236
      N2D = IPASS(11)                                                    WOLFQP    237
      M = N + 1                                                          WOLFQP    238
      IOBJ = M + N                                                       WOLFQP    239
      PHASE1 = .TRUE.                                                    WOLFQP    240
      NP1 = N + 1                                                        WOLFQP    241
      NPM = N + M                                                        WOLFQP    242
      IP1 = NPM + 1                                                      WOLFQP    243
      IP2 = 2*NPM + 1                                                    WOLFQP    244
      IP3 = 3*NPM + 1                                                    WOLFQP    245
      IP4 = 4*NPM + 1                                                    WOLFQP    246
      IP5 = 5*NPM + 1                                                    WOLFQP    247
      IP6 = 5*NPM + N + 1                                                WOLFQP    248
      IP7 = N**2 + 11*N + 6                                              WOLFQP    249
C     CALL SETUP ROUTINE                                                 WOLFQP    250
  100 CALL SETUP(SCR(IP7), OBJ, SCR(1), SCR(IP1), SCR(IP2), SCR(IP3),    WOLFQP    251
     *            SCR(IP4),SCR(IP6),IPASS(5),IND)                        WOLFQP    252
      CALL PSOL(Q,A1,A2,OBJ,IND,SCR(IP7),SCR(IP2),SCR(IP6),NQD,N1D,N2D)  WOLFQP    253
  105 ITER = 0                                                           WOLFQP    254
      ITMAX = IPASS(2)                                                   WOLFQP    255
      NINVT = IPASS(8)                                                   WOLFQP    256
      CALL DSIMP(Q,A1,A2,OBJ,IND,SCR(1),SCR(IP1),SCR(IP7),SCR(IP2),      WOLFQP    257
     *               SCR(IP3),SCR(IP6),SCR(IP5),SCR(IP4),NQD,N1D,N2D)    WOLFQP    258
C                                                                        WOLFQP    259
C                                                                        WOLFQP    260
      IF(IERR .NE. 0) GO TO 200                                          WOLFQP    261
      IF(PHASE1) 110, 120                                                WOLFQP    262
C     THEN CHECK FOR CONSISTENT CONSTRAINTS                              WOLFQP    263
  110    IF(SCR(IP2+IOBJ-1) .LE. 0.) PHASE1 = .FALSE.                    WOLFQP    264
         IF(.NOT.PHASE1) GO TO 100                                       WOLFQP    265
C       ELSE                                                             WOLFQP    266
            IERR = 1                                                     WOLFQP    267
            GO TO 200                                                    WOLFQP    268
C       END                                                              WOLFQP    269
C     ELSE                                                               WOLFQP    270
  120    IF(SCR(IP2+IOBJ-1) .NE. 0.) IERR = 4 + IERR                     WOLFQP    271
         IF(VALUE .EQ. 0.) GO TO 200                                     WOLFQP    272
C       ELSE                                                             WOLFQP    273
            VALUE = 0.                                                   WOLFQP    274
            DO 140 I = 1, N1                                             WOLFQP    275
      SUBROUTINE WOLFQP      73/74    OPT=1                           FTN 4.6+452       82/01/28. 14.34.48       PAGE      6

                                TEMP = 0.                                                WOLFQP    276
                                DO 130 J = 1, N1                                         WOLFQP    277
                                   TEMP = TEMP + Q((J-1)*NQD+I)*SCR(IP3-1+J+N)           WOLFQP    278
                   130          CONTINUE                                                 WOLFQP    279
    290                         VALUE = VALUE + SCR(IP3-1+I+N)*(TEMP/2. + OBJ(I))        WOLFQP    280
                   140       CONTINUE                                                    WOLFQP    281
                 C        END                                                            WOLFQP    282
                 C     END                                                               WOLFQP    283
                   200 CONTINUE                                                          WOLFQP    284
    295                DO 210 J = 1, N1                                                  WOLFQP    285
                          Y(J) = SCR(IP3-1+J+N)                                          WOLFQP    286
                   210 CONTINUE                                                          WOLFQP    287
                   220 CONTINUE                                                          WOLFQP    288
                       IPASS(4) = IERR                                                   WOLFQP    289
    300                IPASS(3) = ITER                                                   WOLFQP    290
                       RETURN                                                            WOLFQP    291
                       END                                                               WOLFQP    292


      SYMBOLIC REFERENCE MAP (R=2)

  ENTRY POINTS     DEF LINE     REFERENCES
      4  WOLFQP         1        301

  VARIABLES      SN  TYPE             RELOCATION
      0  A1          REAL                   F.P.       REFS       263      267   DEFINED        1
      0  A2          REAL                   F.P.       REFS       263      267   DEFINED        1
     20  BIGM        REAL                  XXXQP       REFS       224  DEFINED       234
      0  C           REAL                   F.P.       REFS       284  DEFINED         1
      4  EPS         REAL                  XXXQP       REFS       222  DEFINED       239
      0  EPS1        REAL                   F.P.       REFS       239  DEFINED         1
      0  FLAG        LOGICAL                F.P.       REFS       220      238   DEFINED        1
    337  I           INTEGER                           REFS       288    2*290   DEFINED      285
      6  IERR        INTEGER               XXXQP       REFS       222      271       281      299   DEFINED      277      281
      0  IND         INTEGER     ARRAY      F.P.       REFS       219      261       263      267   DEFINED        1
      0  IOBJ        INTEGER               XXXQP       REFS       222      274       281  DEFINED       249
      0  IPASS       INTEGER     ARRAY      F.P.       REFS       219      240     2*241      243      245      246      247
                                                        261       265      266   DEFINED        1      299      300
     11  IPIV        INTEGER               XXXQP       REFS       223
    330  IP1         INTEGER                           REFS       261      267   DEFINED      253
    331  IP2         INTEGER                           REFS       261      263       267      274      281
                                                    DEFINED       254
    332  IP3         INTEGER                           REFS       261      267       288      290      296
                                                    DEFINED       255
    333  IP4         INTEGER                           REFS       261      267   DEFINED      256
    334  IP5         INTEGER                           REFS       267  DEFINED       257
    335  IP6         INTEGER                           REFS       261      263       267  DEFINED      258
    336  IP7         INTEGER                           REFS       261      263       267  DEFINED      259
      5  ITER        INTEGER               XXXQP       REFS       222      300   DEFINED      264
      3  ITMAX       INTEGER               XXXQP       REFS       222  DEFINED       265
    341  J           INTEGER                           REFS     2*288    2*296   DEFINED      287      295
     12  JPIV        INTEGER               XXXQP       REFS       223
      1  M           INTEGER               XXXQP       REFS       222      249       252  DEFINED      248
      2  N           INTEGER               XXXQP       REFS       222      248       249      251      252      258    2*259
      SUBROUTINE WOLFOP      73/74    OPT=1                        FTN 4.6+452        82/01/28. 14.34.48        PAGE      7

VARIABLES      SN   TYPE            RELOCATION
                                                   288        290        296   DEFINED      240
   13  NEGV       LOGICAL             XXXOP       REFS        223        225
   21  NINVT      INTEGER             XXXOP       REFS        224    DEFINED      266
   10  NPM        INTEGER             XXXOP       REFS        223        253      254          255        256        257        258
                                               DEFINED        252
    7  NP1        INTEGER             XXXOP       REFS        223    DEFINED      251
  325  NUD        INTEGER                         REFS        263        267      288      DEFINED        245
   14  N1         INTEGER             XXXOP       REFS        223        242      243          285        287        295
                                               DEFINED        241
  326  N1D        INTEGER                         REFS        263        267   DEFINED          246
   15  N11        INTEGER             XXXOP       REFS        223    DEFINED      242
   16  N2         INTEGER             XXXOP       REFS        223        244   DEFINED          243
  327  N2D        INTEGER                         REFS        263        267   DEFINED          247
   17  N21        INTEGER             XXXOP       REFS        223    DEFINED      244
    0  OBJ        REAL         ARRAY   F.P.       REFS        219        261      263          267        290
                                               DEFINED          1
   22  PHASE1     LOGICAL             XXXOP       REFS        224        225      272          275    DEFINED        250        274
    0  Q          REAL         ARRAY   F.P.       REFS        219        263      267          288    DEFINED          1
    0  SCR        REAL         ARRAY   F.P.       REFS        219      7*261    3*263        8*267        274        281        288
                                                   290        296    DEFINED        1
  340  TEMP       REAL                            REFS        288        290   DEFINED          286        288
    0  VALUE      REAL                 F.P.       REFS        282        290   DEFINED            1        284        290
    0  Y          REAL         ARRAY   F.P.       REFS        219    DEFINED        1            296

EXTERNALS               TYPE    ARGS        REFERENCES
      DSIMP                      16             267
      PSOL                       11             263
      SETUP                      10             261

STATEMENT LABELS                   DEF LINE    REFERENCES
   54  100                            261         275
  115  105                            264         238
    0  110           INACTIVE         274         272
  164  120                            281         272
    0  130                            289         287
    0  140                            291         285
  232  200                            294         271        278        282
    0  210                            297         295
    0  220           INACTIVE         298

LOOPS  LABEL      INDEX      FROM-TO      LENGTH     PROPERTIES
  176  140      * I          285 291        33B                  NOT INNER
  212  130        J          287 289         4B       INSTACK
  243  210        J          295 297         2B       INSTACK

COMMON BLOCKS   LENGTH
      XXXOP        19

STATISTICS
  PROGRAM LENGTH                    373B      251
  CM LABELED COMMON LENGTH           23B       19
        52000B CM USED
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SUBROUTINE SETUP      73/74   OPT=1                  FTN 4.6+452       82/01/28. 14.34.48       PAGE    1

            SUBROUTINE SETUP(K,OBJ,BL,BU,X,U,BASIC,E,NCON,IND)                   SETUP      2
      C                                                                          SETUP      3
      C                                                                          LINE       2
      C------------------------------------------------------------------------- LINE       3
      C                                                                          LINE       4
            DIMENSION K(1), OBJ(1), BL(1), BU(1), X(1), U(1)                     SETUP      5
            DIMENSION BASIC(1), E(1), IND(1)                                     SETUP      6
            INTEGER BASIC                                                        SETUP      7
      C                                                                          LINE       2
      C------------------------------------------------------------------------- LINE       3
      C                                                                          LINE       4
            COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                    /XXXQP/    2
            COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21          /XXXQP/    3
            COMMON /XXXQP/ HIGH, NINVT, PHASE1                                   /XXXQP/    4
            LOGICAL NEGV, PHASE1                                                 /XXXQP/    5
      C                                                                          LINE       2
      C------------------------------------------------------------------------- LINE       3
      C                                                                          LINE       4
      C                                                                          SETUP     11
      C     INTERNAL COMPOSITION OF FLAG WORD BASIC                              SETUP     12
      C                                                                          SETUP     13
      C        BASIC(*) =    (COMPLEMENTARY INDEX)  ABCDEFG                      SETUP     14
      C                                                                          SETUP     15
      C        COMPLEMENTARY INDEX - BITS(8-60) THIS CONTAINS THE INDEX          SETUP     16
      C                              OF THE COMPLEMENTARY VARIABLE IF            SETUP     17
      C                              COMPLEMENTARY SLACKNESS APPLIES TO          SETUP     18
      C                              THIS VARIABLE.                              SETUP     19
      C                                                                          SETUP     20
      C        A  - BIT(7)   1 - WHEN VARIABLE BECOMES NON-BASIC CHANGE          SETUP     21
      C                          LOWER BOUND TO ZERO, SET PRIMAL VARIABLE        SETUP     22
      C                          TO ZERO, AND SET BIT A TO ZERO IN THIS          SETUP     23
      C                          WORD AND IN IT"S COMPLEMENT FLAG WORD.          SETUP     24
      C                      0 - NO CHANGE WHEN VARIABLE BECOMES NON-BASIC       SETUP     25
      C                                                                          SETUP     26
      C        B  - BIT(6)   1 - WHEN VARIABLE BECOMES NON-BASIC MAINTAIN        SETUP     27
      C                          UPPER BOUND                                     SETUP     28
      C                      0 - CHANGE UPPER BOUND TO +INFINITY WHEN            SETUP     29
      C                          VARIABLE BECOMES NON-BASIC.                     SETUP     30
      C                                                                          SETUP     31
      C        C  - BIT(5)   1 - WHEN VARIABLE BECOMES NON-BASIC MAINTAIN        SETUP     32
      C                          LOWER BOUND.                                    SETUP     33
      C                      0 - WHEN VARIABLE BECOMES NON-BASIC CHANGE          SETUP     34
      C                          LOWER BOUND TO -INFINITY.                       SETUP     35
      C                                                                          SETUP     36
      C        D  - BIT(4)   1 - VARIABLE IS BASIC                               SETUP     37
      C                      0 - VARIABLE IS NON-BASIC                           SETUP     38
      C                                                                          SETUP     39
      C        E  - BIT(3)   1 - COMPLEMENTARY SLACKNESS APPLIES TO VARIBALE     SETUP     40
      C                      0 - COMPLEMENTARY SLACKNESS DOES NOT APPLY          SETUP     41
      C                                                                          SETUP     42
      C        F  - BIT(2)   1 - CHECK UPPER BOUND FOR CONSTRAINT VIOLATIONS     SETUP     43
      C                      0 - DON"T CHECK UPPER BOUND                         SETUP     44
      C                                                                          SETUP     45
      C        G  - BIT(1)   1 - CHECK LOWER BOUND FOR CONSTRAINT VIOLATIONS     SETUP     46
      C                      0 - DON"T CHECK LOWER BOUND                         SETUP     47
      C                                                                          LINE       2
      C------------------------------------------------------------------------- LINE       3
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SUBROUTINE SETQP      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE     2

      C                                                                              LINE         4
              DATA MASK1 / 77B/, MASK2 / 66B/, MASK3 / 73B/, MASK4 / 63B/            SETQP       49
              DATA MASK5 / 76B/, MASK6 / 64B/, MASK7 /70B/, MASK8 /60B/              SETQP       50
              DATA MASK9 / 1B/, MASK10 / 2B/, MASK11 / 10B/, MASK12 / 37B/           SETQP       51
              DATA MASK13 / 117B/, MASK14 / 13B/, MASK15 / 100B/                     SETQP       52
      C                                                                              LINE         2
      C----------------------------------------------------------------------------- LINE         3
      C                                                                              LINE         4
              IF(.NOT.PHASE1) GO TO 65                                               SETQP       54
      C                                                                              SETQP       55
      C          PHASE I                                                             SETQP       56
      C                                                                              SETQP       57
      C                                                                              SETQP       58
      C       SET E TO THE IDENTITY                                                  SETQP       59
      C                                                                              SETQP       60
              JJ = 0                                                                 SETQP       61
              DO 15 J = 1, N                                                         SETQP       62
                 DO 10 I = 1, N                                                      SETQP       63
                    E(I+JJ) = 0.                                                     SETQP       64
         10      CONTINUE                                                            SETQP       65
                 E(J+JJ) = 1.                                                        SETQP       66
                 JJ = JJ + N                                                         SETQP       67
           15 CONTINUE                                                               SETQP       68
      C                                                                              SETQP       69
      C       SET INTERNAL FLAGS(BASIC), UPPER AND LOWER BOUNDS                      SETQP       70
      C       AND INITIAL PRIMAL AND DUAL SOLUTIONS                                  SETQP       71
      C                                                                              SETQP       72
              DO 50 J = 1, N                                                         SETQP       73
                 K(J) = J                                                            SETQP       74
                 IF(J .GT. NCON) GO TO 16                                            SETQP       75
      C         ELSE                                                                 SETQP       76
      C         J   - DUAL TO ARTIFICIAL OR SLACK ON CONSTRAINED VARIABLE            SETQP       77
      C         J+N - DUAL TO CONSTRAINED VARIABLE                                   SETQP       78
      C                                                                              SETQP       79
                    BASIC(IND(J)) = OR(MASK1,SHIFT(IND(J)+N,7))                      SETQP       80
                    BASIC(IND(J)+N) = OR(MASK2,SHIFT(IND(J),7))                      SETQP       81
                    BL(IND(J)) = 0.                                                  SETQP       82
                    BU(IND(J)) = 0.                                                  SETQP       83
                    BL(IND(J)+N) = -BIGM                                             SETQP       84
                    BU(IND(J)+N) = 0.                                                SETQP       85
                    X(IND(J)) = 0.                                                   SETQP       86
                    GO TO 19                                                         SETQP       87
      C         END                                                                  SETQP       88
           16    IF(J .GT. N1) GO TO 17                                              SETQP       89
      C         ELSE                                                                 SETQP       90
      C         J   - DUAL TO ARTIFICIAL ON UNCONSTRAINED VARIABLE                   SETQP       91
      C         J+N - DUAL TO UNCONSTRAINED VARIABLE                                 SETQP       92
      C                                                                              SETQP       93
                    BASIC(IND(J)) = MASK3                                            SETQP       94
                    BASIC(IND(J)+N) = MASK4                                          SETQP       95
                    BU(IND(J)) = 0.                                                  SETQP       96
                    BL(IND(J)) = 0.                                                  SETQP       97
                    BL(IND(J)+N) = 0.                                                SETQP       98
                    BU(IND(J)+N) = 0.                                                SETQP       99
                    X(IND(J)) = 0.                                                   SETQP      100
                    GO TO 19                                                         SETQP      101
      C         END                                                                  SETQP      102
      SUBROUTINE SETUP      73/74    OPT=1                FTN 4.6+452        82/01/28. 14.34.48        PAGE    3

                 17   IF(J .GT. N2) GO TO 18                                           SETUP      103
           C        ELSE                                                               SETUP      104
           C        J   - DUAL TO ARTIFICIAL OR SLACK ON INEQUALITY CONSTRAINTS        SETUP      105
           C        J+N - DUAL TO LAGRANGE MULT ON INEQUALITY CONSTRAINTS              SETUP      106
           C                                                                           SETUP      107
                          BASIC(J) = OR(MASK5,SHIFT(J+N,7))                            SETUP      108
                          BASIC(J+N) = MASK6                                           SETUP      109
                          BL(J) = - BIGM                                               SETUP      110
                          BU(J) = 0.                                                   SETUP      111
                          BL(J+N) = -BIGM                                              SETUP      112
                          BU(J+N) = 0.                                                 SETUP      113
                          X(J) = 0.                                                    SETUP      114
                          GO TO 19                                                     SETUP      115
           C        END                                                                SETUP      116
           C        J   - DUAL TO ARTIFICIAL ON EQUALITY CONSTRAINTS                   SETUP      117
           C        J+N - DUAL TO LAGRANGE MULT ON EQUALITY CONSTRAINTS                SETUP      118
           C                                                                           SETUP      119
                 18    BASIC(J) = MASK7                                                SETUP      120
                       BASIC(J+N) = MASK8                                              SETUP      121
                       BL(J) = -BIGM                                                   SETUP      122
                       BU(J) = BIGM                                                    SETUP      123
                       IF(OBJ(J) .GT. 0.) BASIC(J) = OR(BASIC(J),MASK10)               SETUP      124
                       IF(OBJ(J) .LE. 0.) BASIC(J) = OR(BASIC(J),MASK9)                SETUP      125
                       BL(J+N) = 0.                                                    SETUP      126
                       BU(J+N) = 0.                                                    SETUP      127
                       X(J) = BIGM                                                     SETUP      128
                 19    CONTINUE                                                        SETUP      129
                       KJ = K(J)                                                       SETUP      130
                       IF(J .LE. N1) KJ = IND(J)                                       SETUP      131
                       U(KJ) = OBJ(KJ)                                                 SETUP      132
                       IF(J .LE. N1) U(KJ) = -U(KJ)                                    SETUP      133
                       IF (U(KJ))  20, 40, 30                                          SETUP      134
           C           NEGATIVE                                                        SETUP      135
                 20       CONTINUE                                                     SETUP      136
                          X(KJ) = BL(KJ)                                               SETUP      137
                          GO TO 40                                                     SETUP      138
           C           POSITIVE                                                        SETUP      139
                 30       CONTINUE                                                     SETUP      140
                          X(KJ) = BU(KJ)                                               SETUP      141
                 40    CONTINUE                                                        SETUP      142
                 50    CONTINUE                                                        SETUP      143
                    DO 60 J = NP1, NPM                                                 SETUP      144
                       K(J) = J                                                        SETUP      145
                       U(K(J)) = 0.                                                    SETUP      146
                 60    CONTINUE                                                        SETUP      147
                    BL(IOBJ) = -BIGM                                                   SETUP      148
                    BU(IOBJ) = BIGM                                                    SETUP      149
                    BASIC(IOBJ) = MASK8                                                SETUP      150
                    RETURN                                                             SETUP      151
           C                                                                           SETUP      152
           C                                                                           SETUP      153
           C             SETUP FOR PHASE II                                            SETUP      154
           C                                                                           SETUP      155
           C                                                                           SETUP      156
                 65 DO 110 J = 1, N1                                                   SETUP      157
                         IF(J .GT. NCON) GO TO 90                                      SETUP      158
           C            ELSE                                                           SETUP      159
```

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     SUBROUTINE SETUP      73/74    OPT=1                      FTN 4.6+452       82/01/28. 14.34.48       PAGE     4

               C       CONSTRAINED VARIABLE AND COMPLEMENT                                SETUP     160
               C                                                                          SETUP     161
                          BU(IND(J)) = 1.                                                 SETUP     162
                          L2 = SHIFT(IND(J)+N,7)                                          SETUP     163
                          L1 = AND(BASIC(IND(J)+N),MASK11)                                SETUP     164
                          IF(L1 .EQ. 0) 70, 80                                            SETUP     165
               C         THEN                                                             SETUP     166
               C         VARIABLE IS ARTIFICIAL                                           SETUP     167
                    70        BL(IND(J)) = 0.                                             SETUP     168
                              BASIC(IND(J)) = OR(MASK12,L2)                               SETUP     169
                              GO TO 100                                                   SETUP     170
               C         ELSE                                                             SETUP     171
               C         VARIABLE IS SLACK                                                SETUP     172
                    80        BL(IND(J)) = -1.                                            SETUP     173
                              BASIC(IND(J)) = OR(MASK13,L2)                               SETUP     174
                              BASIC(IND(J)+N) = OR(BASIC(IND(J)+N),MASK15)                SETUP     175
                              GO TO 100                                                   SETUP     176
               C         END                                                              SETUP     177
               C       END                                                                SETUP     178
               C       UNCONSTRAINED VARIABLES                                            SETUP     179
               C                                                                          SETUP     180
                    90   BL(IND(J)) = -1.                                                 SETUP     181
                         BU(IND(J)) = 1.                                                  SETUP     182
                         BASIC(IND(J)) = MASK14                                           SETUP     183
               C                                                                          SETUP     184
               C         SET PRIMAL VARIABLE WITH NEW BOUNDS                              SETUP     185
               C                                                                          SETUP     186
                   100   X(IND(J)) = BL(IND(J))                                           SETUP     187
                         IF(U(IND(J)) .GT. 0.) X(IND(J)) = BU(IND(J))                     SETUP     188
                   110 CONTINUE                                                           SETUP     189
               C                                                                          SETUP     190
               C      SET FLAGS FOR COMPLEMENTS OF EQUALITY CONSTRAINTS                   SETUP     191
               C                                                                          SETUP     192
                       IF(N11 .GT. N2) GO TO 125                                          SETUP     193
                       DO 120 J = N11, N2                                                 SETUP     194
                          BASIC(J+N) = OR(SHIFT(J,7),MASK2)                               SETUP     195
                   120 CONTINUE                                                           SETUP     196
                   125 IF(N21 .GT. N) GO TO 140                                           SETUP     197
                       DO 130 J = N21, N                                                  SETUP     198
                          BASIC(J+N) = MASK4                                              SETUP     199
                   130 CONTINUE                                                           SETUP     200
                   140 CONTINUE                                                           SETUP     201
                       RETURN                                                             SETUP     202
                       END                                                                SETUP     203


     SYMBOLIC REFERENCE MAP (R=2)

 ENTRY POINTS      DEF LINE      REFERENCES
     4  SETUP          1          163       214

 VARIABLES       SN   TYPE           RELOCATION
     0  BASIC       INTEGER      ARRAY      F.P.        REFS          7         8        136       137       176       187
                                                         DEFINED      1        92         93       106       107       120       121
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        SUBROUTINE SETUP      73/74    OPT=1                         FTN 4.6+452       82/01/28. 14.34.48      PAGE     5

   VARIABLES     SN   TYPE              RELOCATION
                                                      132       133       136       137       162       181       186       187
                                                      195       207       211
     20  BIGM        REAL                   XXXQP     REFS       14        96       122       124       134       135       140
                                                      160       161
      0  BL          REAL         ARRAY      F.P.     REFS        6       149       199   DEFINED         1        94        96
                                                      109       110       122       124       134       138       160       180
                                                      185       193
      0  BU          REAL         ARRAY      F.P.     REFS        6       153       200   DEFINED         1        95        97
                                                      108       111       123       125       135       139       161       174
                                                      194
      0  E           REAL         ARRAY      F.P.     REFS        7   DEFINED         1        76        78
      4  EPS         REAL                   XXXQP     REFS       12
    314  I           INTEGER                          REFS       76   DEFINED        75
      6  IERR        INTEGER                XXXQP     REFS       12
      0  IND         INTEGER      ARRAY      F.P.     REFS        7      2*92      2*93        94        95        96        97
                                                       98       106       107       108       109       110       111       112
                                                      143       174       175       176       180       181       185       186
                                                    2*187       193       194       195     2*199     3*200
                                                  DEFINED         1
      0  IOBJ        INTEGER                XXXQP     REFS       12       160       161       162
     11  IPIV        INTEGER                XXXQP     REFS       13
      5  ITER        INTEGER                XXXQP     REFS       12
      3  ITMAX       INTEGER                XXXQP     REFS       12
    313  J           INTEGER                          REFS       78      2*86        87      2*92      2*93        94        95
                                                       96        97        98       101       106       107       108       109
                                                      110       111       112       115     2*120       121       122       123
                                                      124       125       126       132       133       134       135     3*136
                                                    3*137       138       139       140       142     2*143       145     2*157
                                                      158       170       174       175       176       180       181       185
                                                      186     2*187       193       194       195     2*199     3*200     2*207
                                                      211   DEFINED        74        85       156       169       206       210
    312  JJ          INTEGER                          REFS       76        78        79   DEFINED        73        79
     12  JPIV        INTEGER                XXXQP     REFS       13
      0  K           INTEGER      ARRAY      F.P.     REFS        6       142       158   DEFINED         1        86       157
    315  KJ          INTEGER                          REFS    2*144     2*145       146     2*149     2*153
                                                  DEFINED       142       143
    317  L1          INTEGER                          REFS      177   DEFINED       176
    316  L2          INTEGER                          REFS      181       186   DEFINED       175
      1  M           INTEGER                XXXQP     REFS       12
    271  MASK1       INTEGER                          REFS       92   DEFINED        59
    302  MASK10      INTEGER                          REFS      136   DEFINED        61
    303  MASK11      INTEGER                          REFS      176   DEFINED        61
    304  MASK12      INTEGER                          REFS      181   DEFINED        61
    305  MASK13      INTEGER                          REFS      186   DEFINED        62
    306  MASK14      INTEGER                          REFS      195   DEFINED        62
    307  MASK15      INTEGER                          REFS      187   DEFINED        62
    272  MASK2       INTEGER                          REFS       93       207   DEFINED        59
    273  MASK3       INTEGER                          REFS      106   DEFINED        59
    274  MASK4       INTEGER                          REFS      107       211   DEFINED        59
    275  MASK5       INTEGER                          REFS      120   DEFINED        60
    276  MASK6       INTEGER                          REFS      121   DEFINED        60
    277  MASK7       INTEGER                          REFS      132   DEFINED        60
    300  MASK8       INTEGER                          REFS      133       162   DEFINED        60
    301  MASK9       INTEGER                          REFS      137   DEFINED        61
      2  N           INTEGER                XXXQP     REFS       12        74        75        79        85        92        93
                                                       96        97       107       110       111       120       121       124
                                                      125       133       138       139       175       176     2*187       207
      SUBROUTINE SETUP      73/74   OPT=1                         FTN 4.6+452       82/01/28. 14.34.48      PAGE     6

VARIABLES     SN  TYPE            RELOCATION
                                                 209       210       211
    0  NCON       INTEGER                F.P.    REFS       87       170    DEFINED       1
   13  NEGV       LOGICAL                XXXUP   REFS       13        15
   21  NINVT      INTEGER                XXXUP   REFS       14
   10  NPM        INTEGER                XXXUP   REFS       13       156
    7  NP1        INTEGER                XXXUP   REFS       13       156
   14  N1         INTEGER                XXXUP   REFS       13       101       143       145       169
   15  N11        INTEGER                XXXUP   REFS       13       205       206
   16  N2         INTEGER                XXXUP   REFS       13       115       205       206
   17  N21        INTEGER                XXXUP   REFS       13       209       210
    0  OBJ        REAL        ARRAY      F.P.    REFS        6       136       137       144    DEFINED       1
   22  PHASE1     LOGICAL                XXXUP   REFS       14        15        66
    0  U          REAL        ARRAY      F.P.    REFS        6       145       146       200    DEFINED       1        144
                                                 145       158
    0  X          REAL        ARRAY      F.P.    REFS        6    DEFINED        1        98       112       126      140
                                                 149       153       199       200

INLINE FUNCTIONS    TYPE    ARGS         DEF LINE  REFERENCES
       AND        NO TYPE    0  INTRIN               176
       OR *       NO TYPE    0  INTRIN                92        93       120       136       137       181       186      187
                                                     207
       SHIFT      NO TYPE    2  INTRIN                92        93       120       175       207

STATEMENT LABELS                   DEF LINE    REFERENCES
    0  10                              77         75
    0  15                              80         74
   67  16                             101         87
   76  17                             115        101
  106  18                             132        115
  122  19                             141         99       113       127
    0  20            INACTIVE         148        146
  140  30                             152        146
  142  40                             154        146       150
    0  50                             155         85
    0  60                             159        156
  167  65                             169         66
    0  70            INACTIVE         180        177
  212  80                             185        177
  220  90                             193        170
  224  100                            199        182       188
    0  110                            201        169
    0  120                            208        206
  251  125                            209        205
    0  130                            212        210
  266  140                            213        209

LOOPS  LABEL      INDEX      FROM-TO      LENGTH     PROPERTIES
   23  15       * J           74 80        20B                  NOT INNER
   32  10         I           75 77         2B        INSTACK
   56  50         J           85 155       66B          OPT
  154  60         J          156 159        2B        INSTACK
  200  110        J          169 201       31B          OPT
  244  120        J          206 208        3B        INSTACK
  262  130        J          210 212        2B        INSTACK
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      SUBROUTINE SETUP      73/74    OPT=1                          FTN 4.6+452       82/01/28. 14.34.48       PAGE     7

COMMON BLOCKS   LENGTH
      XXXOP         19

STATISTICS
  PROGRAM LENGTH                 362B       242
  CM LABELED COMMON LENGTH        23B        19
         52000B CM USED
```

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SUBROUTINE DSIMP      73/74    OPT=1                  FTN 4.6+452        82/01/28. 14.34.48        PAGE     1

            SUBROUTINE DSIMP(Q,A1,A2,OBJ,IND,BL,BU,K,X,U,E,AROW,BASIC,NQD,NID, DSIMP      2
           *                 N2D)                                               DSIMP      3
      C                                                                         LINE       2
      C------------------------------------------------------------------------ LINE       3
      C                                                                         LINE       4
      C     DUAL SIMPLEX ALGORITHM                                              DSIMP      5
      C                                                                         LINE       2
      C------------------------------------------------------------------------ LINE       3
      C                                                                         LINE       4
            DIMENSION BL(1), BU(1), K(1), X(1), U(1), AROW(1), BASIC(1)         DSIMP      7
            INTEGER BASIC                                                       DSIMP      8
      C                                                                         DSIMP      9
            COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                   /XXXQP/    2
            COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21         /XXXQP/    3
            COMMON /XXXQP/ BIGM, NINVT, PHASE1                                  /XXXQP/    4
            LOGICAL NEGV, PHASE1                                                /XXXQP/    5
      C                                                                         LINE       2
      C------------------------------------------------------------------------ LINE       3
      C                                                                         LINE       4
            DATA MASK1 /10B/, MASK2 /7B/, MASK3 /60B/, MASK4 /100B/             DSIMP     12
      C                                                                         DSIMP     13
            IERR = 2                                                            DSIMP     14
      C     WHEN ITMAX.LT.1 THE LOOP IS PARTIALLY EXECUTED                      DSIMP     15
            DO 100 II = 1, ITMAX                                                DSIMP     16
               CALL PIVROW(BL,BU,K,X,U,BASIC)                                   DSIMP     17
               IF (IPIV.GT.0)  GO TO 10                                         DSIMP     18
      C           NO PIVOT ROW INDICATES THAT X IS OPTIMAL                      DSIMP     19
                  IERR = 0                                                      DSIMP     20
                  RETURN                                                        DSIMP     21
         10    CONTINUE                                                         DSIMP     22
               KROW = K(IPIV)                                                   DSIMP     23
               CALL GETROW(Q,A1,A2,OBJ,IND,E,KROW,AROW,NQD,NID,N2D)             DSIMP     24
               CALL PIVCOL(BL,BU,K,X,U,AROW)                                    DSIMP     25
               IF (JPIV.GT.0)  GO TO 40                                         DSIMP     26
      C           NO PIVOT COLUMN INDICATES THAT THE CONSTRAINTS                DSIMP     27
      C           ARE INCONSISTENT                                              DSIMP     28
                  IERR = 1                                                      DSIMP     29
                  RETURN                                                        DSIMP     30
         40    CONTINUE                                                         DSIMP     31
               IF (ITMAX.LT.1)  RETURN                                          DSIMP     32
      C        NEW SOLUTION KEY                                                 DSIMP     33
               KK = K(JPIV)                                                     DSIMP     34
      C                                                                         DSIMP     35
      C        KROW IS NEW BASIC VARIABLE  KK IS EXITING VARIABLE               DSIMP     36
      C        SET BITS IN BASIC FLAG WORDS APPROPRIATELY                       DSIMP     37
      C                                                                         DSIMP     38
               BASIC(KROW) = OR(BASIC(KROW),MASK1)                              DSIMP     39
               BASIC(KK) = AND(BASIC(KK),COMPL(MASK1))                          DSIMP     40
      C                                                                         DSIMP     41
      C        CHECK FOR CHANGES IN BOUNDS FOR EXITING VARIABLE                 DSIMP     42
      C        MAINT = 3 INDICATES NO CHANGE                                    DSIMP     43
      C                                                                         DSIMP     44
               MAINT = AND(SHIFT(BASIC(KK),-4),MASK2)                           DSIMP     45
               IF(MAINT .EQ. 3) GO TO 50                                        DSIMP     46
      C       ELSE                                                              DSIMP     47
      C                                                                         DSIMP     48
      C           MAINT LESS THAN 3 INDICATES BOUNDS CHANGE                     DSIMP     49
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SUBROUTINE DSIMP       73/74    OPT=1                        FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

          C          TO + OR - INFINITY                                                DSIMP       50
          C                                                                            DSIMP       51
                     IF(MAINT .LT. 3) GO TO 45                                         DSIMP       52
          C         ELSE                                                               DSIMP       53
          C                                                                            DSIMP       54
          C            MAINT GREATER THAN 3 INDICATES LOWER BOUND                      DSIMP       55
          C            CHANGES TO ZERO.  CHANGE BOUND, RESET VARIABLE                  DSIMP       56
          C            AND TURN FLAG OFF IN THIS WORD AND IN THE                       DSIMP       57
          C            COMPLEMENTARY FLAG WORD.                                        DSIMP       58
          C                                                                            DSIMP       59
                       JJ = KK                                                         DSIMP       60
                       ICOMP = SHIFT(BASIC(KK),-7)                                     DSIMP       61
                       IF(MAINT .NE. 4) JJ = ICOMP                                     DSIMP       62
                       BASIC(KK) = AND(BASIC(KK),COMPL(MASK4))                         DSIMP       63
                       BASIC(ICOMP) = AND(BASIC(ICOMP),COMPL(MASK4))                   DSIMP       64
                       BL(JJ) = 0.                                                     DSIMP       65
                       IF(X(JJ) .EQ. -1.) X(JJ) = 0.                                   DSIMP       66
                       IF(MAINT .NE. 4) GO TO 50                                       DSIMP       67
                       MAINT = 1                                                       DSIMP       68
          C         END                                                                DSIMP       69
              45     CONTINUE                                                          DSIMP       70
                     BASIC(KK) = OR(AND(BASIC(KK),MAINT),MASK3)                        DSIMP       71
                     IF(MAINT .LT. 2) BU(KK) = BIGM                                    DSIMP       72
                     IF(MAINT .NE. 1) BL(KK) = -BIGM                                   DSIMP       73
          C       END                                                                  DSIMP       74
              50   CONTINUE                                                            DSIMP       75
                   K(IPIV) = KK                                                        DSIMP       76
                   K(JPIV) = KROW                                                      DSIMP       77
                   CALL NEWINV(E,AROW)                                                 DSIMP       78
          C        NEW DUAL SOLUTION                                                   DSIMP       79
                   CALL GETROW(Q,A1,A2,OBJ,IND,E,IOBJ,AROW,NQD,N1D,N2D)                DSIMP       80
                   DO 70 J = 1, N                                                      DSIMP       81
                     U(K(J)) = AROW(J)                                                 DSIMP       82
              70     CONTINUE                                                          DSIMP       83
                   U(K(IPIV)) = 0.                                                     DSIMP       84
          C        NEW PRIMAL SOLUTION                                                 DSIMP       85
                   X(KROW) = BU(KROW)                                                  DSIMP       86
                   IF (NEGV)  X(KROW) = BL(KROW)                                       DSIMP       87
                   CALL PSOL(Q,A1,A2,OBJ,IND,K,X,E,NQD,N1D,N2D)                        DSIMP       88
                   ITER = ITER + 1                                                     DSIMP       89
                   IF(MOD(ITER,NINVT).EQ.0)                                            DSIMP       90
               *            CALL SETINV(Q,A1,A2,OBJ,IND,K,E,AROW,NQD,N1D,N2D)          DSIMP       91
                   IF(IERR .EQ. 3) RETURN                                              DSIMP       92
             100   CONTINUE                                                            DSIMP       93
                 RETURN                                                                DSIMP       94
                 END                                                                   DSIMP       95


    SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS     DEF LINE     REFERENCES
  4  DSIMP           1           29          38          40          100          102
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      SUBROUTINE DSIMP      73/74    OPT=1                        FTN 4.6+452        82/01/28. 14.34.48       PAGE     3

VARIABLES     SN  TYPE             RELOCATION
    0  AROW      REAL       ARRAY    F.P.       REFS      10      32      33      86      88      90      98
                                             DEFINED       1
    0  A1        REAL                F.P.       REFS      32      88      96      98 DEFINED       1
    0  A2        REAL                F.P.       REFS      32      88      96      98 DEFINED       1
    0  BASIC     INTEGER    ARRAY    F.P.       REFS      10      11      25      47      48      53      69
                                                  71      12      79 DEFINED       1      47      48      71
                                                  72      79
   20  BIGM      REAL                XXXQP      REFS      15      80      81
    0  BL        REAL       ARRAY    F.P.       REFS      10      25      33      95 DEFINED       1      73
                                                  81
    0  BU        REAL       ARRAY    F.P.       REFS      10      25      33      94 DEFINED       1      80
    0  F         REAL                F.P.       REFS      32      86      88      96      98
                                             DEFINED       1
    4  EPS       REAL                XXXQP      REFS      13
  362  ICOMP     INTEGER                        REFS      70    2*72 DEFINED      69
    6  IERR      INTEGER             XXXQP      REFS      13     100 DEFINED      22      28      37
  355  II      * INTEGER                     DEFINED      24
    0  IND       INTEGER             F.P.       REFS      32      88      96      98 DEFINED       1
    0  IOBJ      INTEGER             XXXQP      REFS      13      88
   11  IPIV      INTEGER             XXXQP      REFS      14      26      31      84      92
    5  ITER      INTEGER             XXXQP      REFS      13      97      98 DEFINED      97
    3  ITMAX     INTEGER             XXXQP      REFS      13      24      40
  363  J         INTEGER                        REFS    2*90 DEFINED      89
  361  JJ        INTEGER                        REFS      73    2*74 DEFINED      68      70
   12  JPIV      INTEGER             XXXQP      REFS      14      34      42      85
    0  K         INTEGER    ARRAY    F.P.       REFS      10      25      31      33      42      90      92
                                                  96      98 DEFINED       1      84      85
  357  KK        INTEGER                        REFS    2*48      53      68      69    2*71    2*79      80
                                                  81      84 DEFINED      42
  356  KROW      INTEGER                        REFS      32    2*47      85    2*94    2*95
                                             DEFINED      31
    1  M         INTEGER             XXXQP      REFS      13
  360  MAINT     INTEGER                        REFS      54      60      70      75      79      80      81
                                             DEFINED      53      76
  347  MASK1     INTEGER                        REFS      47      48 DEFINED      20
  350  MASK2     INTEGER                        REFS      53 DEFINED      20
  351  MASK3     INTEGER                        REFS      79 DEFINED      20
  352  MASK4     INTEGER                        REFS      71      72 DEFINED      20
    2  N         INTEGER             XXXQP      REFS      13      89
   13  NEGV      LOGICAL             XXXQP      REFS      14      16      95
   21  NINVT     INTEGER             XXXQP      REFS      15      98
   10  NPM       INTEGER             XXXQP      REFS      14
    7  NP1       INTEGER             XXXQP      REFS      14
    0  NUD       INTEGER             F.P.       REFS      32      88      96      98 DEFINED       1
   14  N1        INTEGER             XXXQP      REFS      14
    0  N1D       INTEGER             F.P.       REFS      32      88      96      98 DEFINED       1
   15  N11       INTEGER             XXXQP      REFS      14
   16  N2        INTEGER             XXXQP      REFS      14
    0  N2D       INTEGER             F.P.       REFS      32      88      96      98 DEFINED       1
   17  N21       INTEGER             XXXQP      REFS      14
    0  OBJ       REAL                F.P.       REFS      32      88      96      98 DEFINED       1
   22  PHASE1    LOGICAL             XXXQP      REFS      15      16
    0  Q         REAL                F.P.       REFS      32      88      96      98 DEFINED       1
    0  U         REAL       ARRAY    F.P.       REFS      10      25      33 DEFINED       1      90      92
    0  X         REAL       ARRAY    F.P.       REFS      10      25      33      74      96
                                             DEFINED       1      74      94      95
      SUBROUTINE DSINP       73/74    OPT=1                          FTN 4.6+452       82/01/28. 14.34.48       PAGE     4

EXTERNALS            TYPE   ARGS         REFERENCES
       GETROW                11             32        88
       NEWINV                 2             86
       PIVCOL                 6             33
       PIVROW                 6             25
       PSOL                  11             96
       SETINV                11             98

INLINE FUNCTIONS   TYPE    ARGS          DEF LINE  REFERENCES
       AND       NO TYPE     0  INTRIN                48        53        71        72        79
       COMPL     NO TYPE     1  INTRIN                48        71        72
       MOD       INTEGER     2  INTRIN                98
       OR        NO TYPE     0  INTRIN                47        79
       SHIFT     NO TYPE     2  INTRIN                53        69

STATEMENT LABELS               DEF LINE    REFERENCES
   36  10                          30         26
   70  40                          39         34
  122  45                          78         60
  136  50                          83         54        75
    0  70                          91         89
    0  100                        101         24

LOOPS  LABEL    INDEX      FROM-TO    LENGTH    PROPERTIES
   22  100    * II          24 101     2234               EXT REFS  NOT INNER
  167  70       J           89 91        38     INSTACK

COMMON BLOCKS   LENGTH
       XXXOP       19

STATISTICS
  PROGRAM LENGTH                 522B     338
  CM LABELED COMMON LENGTH        23B      19
         52000B CM USED
```

```
SUBROUTINE PIVROW      73/74    OPT=1                        FTN 4.6+452         82/01/28. 14.34.48          PAGE      1

              SUBROUTINE PIVROW(BL,BU,K,X,U,BASIC)                                       PIVROW      2
        C                                                                                LINE        2
        C------------------------------------------------------------------------------- LINE        3
        C                                                                                LINE        4
        C     PIVOT ROW SELECTION                                                        PIVROW      4
        C                                                                                LINE        2
        C------------------------------------------------------------------------------- LINE        3
        C                                                                                LINE        4
              DIMENSION BL(1), BU(1), K(1), X(1)                                         PIVROW      6
              DIMENSION BASIC(1)                                                         PIVROW      7
              INTEGER BASIC                                                              PIVROW      8
              DIMENSION U(1)                                                             PIVROW      9
        C                                                                                PIVROW     10
              COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                          /XXXQP/     2
              COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21                /XXXQP/     3
              COMMON /XXXQP/ BIGM, NINVT, PHASE1                                         /XXXQP/     4
              LOGICAL NEGV, PHASE1                                                       /XXXQP/     5
        C                                                                                PIVROW     12
              LOGICAL FLAGU, FLAGL, FLAGB, FLAGC                                         PIVROW     13
        C                                                                                PIVROW     14
        C        THIS ROUTINE HAS BEEN MODIFIED TO HANDLE COMPLEMENTARY                  PIVROW     15
        C        SLACKNESS.  IF COMPLEMENTARY SLACKNESS APPLIES, THE                     PIVROW     16
        C        PIVOT ROW IS REJECTED IF IT"S COMPLEMENT IS BASIC AND                   PIVROW     17
        C        THE PRIMAL VARIABLE IS ZERO.                                            PIVROW     18
        C                                                                                PIVROW     19
        C        FLAGU - CHECK UPPER BOUND FOR CONSTRAINT VIOLATIONS                     PIVROW     20
        C                                                                                PIVROW     21
        C        FLAGL - CHECK LOWER BOUND FOR CONSTRAINT VIOLATIONS                     PIVROW     22
        C                                                                                PIVROW     23
        C        FLAGC - VARIABLE HAS COMPLEMENTARY SLACKNESS CONDITION                  PIVROW     24
        C                                                                                PIVROW     25
        C        FLAGB - COMPLEMENTARY VARIABLE IS BASIC                                 PIVROW     26
        C                                                                                LINE        2
        C------------------------------------------------------------------------------- LINE        3
        C                                                                                LINE        4
              DATA MASK1 /3B/, MASK2 /4B/, MASK3 /10B/                                   PIVROW     28
        C                                                                                PIVROW     29
              IPIV = 0                                                                   PIVROW     30
              IF (NPM.LT.NP1)  RETURN                                                    PIVROW     31
              VIOL = 0.                                                                  PIVROW     32
              DO 50 II = NP1, NPM                                                        PIVROW     33
                 I = K(II)                                                               PIVROW     34
                 L = AND(BASIC(I),MASK1)                                                 PIVROW     35
                 IF(L .EQ. 0) GO TO 50                                                   PIVROW     36
        C       ELSE                                                                     PIVROW     37
                    FLAGU = .FALSE.                                                      PIVROW     38
                    FLAGL = .FALSE.                                                      PIVROW     39
                    IF(L .GT. 1) FLAGU = .TRUE.                                          PIVROW     40
                    IF(L .NE. 2) FLAGL = .TRUE.                                          PIVROW     41
                    FLAGC = .FALSE.                                                      PIVROW     42
                    IF(PHASE1) GO TO 1                                                   PIVROW     43
                    FLAGB = .FALSE.                                                      PIVROW     44
                    L1 = AND(BASIC(I),MASK2)                                             PIVROW     45
                    IF(L1 .EQ. 0) GO TO 1                                                PIVROW     46
        C          ELSE                                                                  PIVROW     47
                       FLAGC = .TRUE.                                                    PIVROW     48
                       ICOMP = SHIFT(BASIC(I),-7)                                        PIVROW     49
```

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    SUBROUTINE PIVROW      73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

                             LI = AND(BASIC(ICOMP),MASK3)                                   PIVROW      50
                             IF(LI .NE. 0) FLAGB = .TRUE.                                   PIVROW      51
 60          C           END                                                                PIVROW      52
             C         END                                                                  PIVROW      53
                 1     CONTINUE                                                             PIVROW      54
             C         CHECK CONSTRAINTS ON X(I)                                            PIVROW      55
                         IF(.NOT.FLAGL) GO TO 10                                            PIVROW      56
 65                      D = X(I) - BL(I)                                                   PIVROW      57
                         IF (D.GE.-EPS)  GO TO 10                                           PIVROW      58
                           D = -D                                                           PIVROW      59
                           IF (VIOL.GT.D)  GO TO 40                                         PIVROW      60
                           IF(.NOT.FLAGC) GO TO 5                                           PIVROW      61
 70          C            ELSE                                                              PIVROW      62
                              IF(FLAGB.AND.X(ICOMP).EQ.0.) GO TO 40                         PIVROW      63
             C            END                                                               PIVROW      64
                 5         CONTINUE                                                         PIVROW      65
                           VIOL = D                                                         PIVROW      66
 75                        IPIV = II                                                        PIVROW      67
                           NEGV = .TRUE.                                                    PIVROW      68
                           GO TO 40                                                         PIVROW      69
                10       CONTINUE                                                           PIVROW      70
                         IF(.NOT.FLAGU) GO TO 40                                            PIVROW      71
 80                      D = X(I) - BU(I)                                                   PIVROW      72
                         IF (D.LE.EPS)  GO TO 30                                            PIVROW      73
                           IF (VIOL.GT.D)  GO TO 40                                         PIVROW      74
                           IF(.NOT.FLAGC) GO TO 20                                          PIVROW      75
             C            ELSE                                                              PIVROW      76
 85                           IF(FLAGB.AND.X(ICOMP).EQ.0.) GO TO 40                         PIVROW      77
             C            END                                                               PIVROW      78
                20         CONTINUE                                                         PIVROW      79
                           VIOL = D                                                         PIVROW      80
                           IPIV = II                                                        PIVROW      81
 90                        NEGV =.FALSE.                                                    PIVROW      82
                30       CONTINUE                                                           PIVROW      83
                40     CONTINUE                                                             PIVROW      84
                50     CONTINUE                                                             PIVROW      85
                     RETURN                                                                 PIVROW      86
 95                  END                                                                    PIVROW      87


    SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
   4  PIVROW          1            39          94

VARIABLES      SN  TYPE             RELOCATION
    0  BASIC       INTEGER      ARRAY     F.P.       REFS         10          11          43          53          57          58
                                                  DEFINED          1
   20  BIGM        REAL                   XXXOP      REFS         16
    0  BL          REAL         ARRAY     F.P.       REFS          9          65    DEFINED          1
    0  BU          REAL         ARRAY     F.P.       REFS          9          80    DEFINED          1
  122  D           REAL                              REFS         66          67          68          74          81          82          88
                                                  DEFINED         65          67          80
    4  EPS         REAL                   XXXOP      REFS         14          66          81
```

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      SUBROUTINE PIVROW      73/74    OPT=1                               FTN 4.6+452       82/01/28. 14.34.48       PAGE      3

VARIABLES      SN  TYPE             RELOCATION
  112  FLAGB      LOGICAL                                REFS         19         71         85    DEFINED        52         59
  113  FLAGC      LOGICAL                                REFS         19         69         83    DEFINED        50         56
  111  FLAGL      LOGICAL                                REFS         19         64    DEFINED         47        49
  110  FLAGU      LOGICAL                                REFS         19         79    DEFINED         46        48
  116  I          INTEGER                                REFS         43         53         57       2*65      2*80
                                                      DEFINED         42
  121  ICOMP      INTEGER                                REFS         58         71         85    DEFINED        57
    6  IERR       INTEGER                XXXUP           REFS         14
  115  II         INTEGER                                REFS         42         75         89    DEFINED        41
    0  IOBJ       INTEGER                XXXUP           REFS         14
   11  IPIV       INTEGER                XXXUP           REFS         15    DEFINED         38         75        89
    5  ITER       INTEGER                XXXUP           REFS         14
    3  ITMAX      INTEGER                XXXUP           REFS         14
   12  JPIV       INTEGER                XXXUP           REFS         15
    0  K          INTEGER    ARRAY       F.P.            REFS          9         42    DEFINED          1
  117  L          INTEGER                                REFS         44         48         49    DEFINED        43
  120  L1         INTEGER                                REFS         54         59    DEFINED         53        58
    1  M .        INTEGER                XXXUP           REFS         14
  105  MASK1      INTEGER                                REFS         43    DEFINED         36
  106  MASK2      INTEGER                                REFS         53    DEFINED         36
  107  MASK3      INTEGER                                REFS         58    DEFINED         36
    2  N          INTEGER                XXXUP           REFS         14
   13  NEGV       LOGICAL                XXXUP           REFS         15         17    DEFINED         76        90
   21  NINVT      INTEGER                XXXUP           REFS         16
   10  NPM        INTEGER                XXXUP           REFS         15         39         41
    7  NP1        INTEGER                XXXUP           REFS         15         39         41
   14  N1         INTEGER                XXXUP           REFS         15
   15  N11        INTEGER                XXXUP           REFS         15
   16  N2         INTEGER                XXXUP           REFS         15
   17  N21        INTEGER                XXXUP           REFS         15
   22  PHASE1     LOGICAL                XXXUP           REFS         16         17         51
    0  U          REAL       ARRAY       F.P.            REFS         12    DEFINED          1
  114  VIOL       REAL                                   REFS         68         82    DEFINED         40        74         88
    0  X          REAL       ARRAY       F.P.            REFS          9         65         71         80        85
                                                      DEFINED          1

INLINE FUNCTIONS    TYPE    ARGS          DEF LINE   REFERENCES
       AND        NO TYPE     0  INTRIN                  43         53         58
       SHIFT      NO TYPE     2  INTRIN                  57

STATEMENT LABELS                  DEF LINE    REFERENCES
   46  1                             62          51         54
   61  5                             73          69
   65  10                            78          64         66
   77  20                            87          83
  102  30                            91          81
  102  40                            92          68         71         77         79         82         85
  102  50                            93          41         44

LOOPS  LABEL      INDEX       FROM-TO      LENGTH      PROPERTIES
   23  50         II            41 93        60B           OPT

COMMON BLOCKS   LENGTH
       XXXUP        19
```

```
        SUBROUTINE PIVROW     73/74   OPT=1                         FTN 4.6+452        82/01/28. 14.34.48        PAGE     4

STATISTICS
  PROGRAM LENGTH                     123B       83
  CM LABELLED COMMON LENGTH           23B       19
         52000B CM USED
```

```
      SUBROUTINE PIVCOL      73/74    OPT=1                              FTN 4.6+452        82/01/28. 14.34.48        PAGE      1

   1              SUBROUTINE PIVCOL(BL,BU,K,X,U,AROW)                                                   PIVCOL       2
            C                                                                                          LINE         2
            C------------------------------------------------------------------------------------ LINE         3
            C                                                                                          LINE         4
   5        C     PIVOT COLUMN SELECTION                                                               PIVCOL       4
            C                                                                                          LINE         2
            C------------------------------------------------------------------------------------ LINE         3
            C                                                                                          LINE         4
                  DIMENSION BL(1), BU(1), K(1), X(1), U(1), AROW(1)                                   PIVCOL       6
  10        C                                                                                          PIVCOL       7
                  COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                                    /XXXQP/      2
                  COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21                          /XXXQP/      3
                  COMMON /XXXQP/ BIGM, NINVT, PHASE1                                                   /XXXQP/      4
                  LOGICAL NEGV, PHASE1                                                                 /XXXQP/      5
  15        C                                                                                          LINE         2
            C------------------------------------------------------------------------------------ LINE         3
            C                                                                                          LINE         4
                  JPIV = 0                                                                             PIVCOL      10
                  W = BIGM                                                                             PIVCOL      11
  20              DO 30 JJ = 1, N                                                                      PIVCOL      12
                     J = K(JJ)                                                                         PIVCOL      13
                     AA = AROW(JJ)                                                                     PIVCOL      14
                     IF (NEGV)  AA = -AA                                                               PIVCOL      15
                     IF (AA.GE.0. .AND. X(J).EQ.BL(J))  GO TO 20                                       PIVCOL      16
  25                 IF (AA.LE.0. .AND. X(J).EQ.BU(J))  GO TO 20                                       PIVCOL      17
                        R = U(J)/AA                                                                    PIVCOL      18
                        IF (R.GT.W)  GO TO 10                                                          PIVCOL      19
                        IF (R.EQ.W .AND. RANF(AA).GT.0.5)  GO TO 10                                    PIVCOL      20
                           JPIV = JJ                                                                   PIVCOL      21
  30                         W = R                                                                     PIVCOL      22
               10       CONTINUE                                                                       PIVCOL      23
               20    CONTINUE                                                                          PIVCOL      24
               30    CONTINUE                                                                          PIVCOL      25
                  RETURN                                                                               PIVCOL      26
  35              END                                                                                  PIVCOL      27


       SYMBOLIC REFERENCE MAP (R=2)

  ENTRY POINTS       DEF LINE       REFERENCES
     4  PIVCOL          1             34

  VARIABLES         SN  TYPE                 RELOCATION
     66  AA             REAL                                     REFS        23         24         25         26         28
                                                              DEFINED        22         23
      0  AROW           REAL         ARRAY       F.P.            REFS         9         22      DEFINED        1
     20  BIGM           REAL                     XXXQP           REFS        13         19
      0  BL             REAL         ARRAY       F.P.            REFS         9         24      DEFINED        1
      0  BU             REAL         ARRAY       F.P.            REFS         9         25      DEFINED        1
      4  EPS            REAL                     XXXQP           REFS        11
      6  IERR           INTEGER                  XXXQP           REFS        11
      0  IOBJ           INTEGER                  XXXQP           REFS        11
     11  IPIV           INTEGER                  XXXQP           REFS        12
      5  ITER           INTEGER                  XXXQP           REFS        11
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                                                                     FTN 4.6+452        82/01/28. 14.34.48          PAGE     2
     SUBROUTINE PIVCOL        73/74    OPT=1

VARIABLES       SN   TYPE                RELOCATION
    3  IIMAX         INTEGER                XXXUP     REFS       11
   65  J             INTEGER                          REFS     2*24      2*25       26      DEFINED      21
   64  JJ            INTEGER                          REFS       21        22       29      DEFINED      20
   12  JPIV          INTEGER                XXXUP     REFS       12   DEFINED       18           29
    0  K             INTEGER      ARRAY      F.P.     REFS        9        21  DEFINED            1
    1  M             INTEGER                XXXUP     REFS       11
    2  N             INTEGER                XXXUP     REFS       11        20
   13  NEGV          LOGICAL                XXXUP     REFS       12        14       23
   21  NINVT         INTEGER                XXXUP     REFS       13
   10  NPM           INTEGER                XXXQP     REFS       12
    7  NP1           INTEGER                XXXUP     REFS       12
   14  N1            INTEGER                XXXUP     REFS       12
   15  N11           INTEGER                XXXQP     REFS       12
   16  N2            INTEGER                XXXQP     REFS       12
   17  N21           INTEGER                XXXQP     REFS       12
   22  PHASEI        LOGICAL                XXXUP     REFS       13        14
   67  R             REAL                             REFS       27        28       30      DEFINED      26
    0  U             REAL         ARRAY      F.P.     REFS        9        26  DEFINED            1
   63  W             REAL                             REFS       27        28  DEFINED           19      30
    0  X             REAL         ARRAY      F.P.     REFS        9        24       25      DEFINED       1

 INLINE FUNCTIONS    TYPE    ARGS           DEF LINE   REFERENCES
        RANF         REAL      1   INTRIN                    28

 STATEMENT LABELS                      DEF LINE    REFERENCES
    56   10                               31          27          28
    56   20                               32          24          25
     0   30                               33          20

 LOOPS  LABEL     INDEX      FROM-TO     LENGTH    PROPERTIES
    33   30       JJ           20 33       24H          OPT

 COMMON BLOCKS    LENGTH
        XXXUP         19

 STATISTICS
   PROGRAM LENGTH                          104B      68
   CM LABELLED COMMON LENGTH                23B      19
          52000B CM USED
```

```
SUBROUTINE GETROW      73/74    OPT=1                      FTN 4.6+452        82/01/28. 14.34.48        PAGE     1

            SUBROUTINE GETROW(Q,A1,A2,OBJ,IND,E,KROW,AROW,NQD,N1D,N2D)          GETROW       2
      C                                                                         LINE         2
      C-------------------------------------------------------------------------LINE         3
      C                                                                         LINE         4
      C     GENERATION OF CONSTRAINT COEFFICIENTS FOR THE CURRENT BASIS         GETROW       4
      C                                                                         LINE         2
      C-------------------------------------------------------------------------LINE         3
      C                                                                         LINE         4
            DIMENSION Q(NQD,1), A1(N1D,1), A2(N2D,1), E(1), AROW(1), OBJ(1)     GETROW       6
            DIMENSION IND(1)                                                    GETROW       7
      C                                                                         GETROW       8
            COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                   /XXXQP/      2
            COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21         /XXXQP/      3
            COMMON /XXXQP/ BIGM, NINVT, PHASE1                                  /XXXQP/      4
            LOGICAL NEGV, PHASE1                                                /XXXQP/      5
      C                                                                         LINE         2
      C-------------------------------------------------------------------------LINE         3
      C                                                                         LINE         4
            IF (KROW.GT.N)  GO TO 20                                            GETROW      11
      C        ORIGINAL INDEPENDENT VARIABLE.  GET ROW KROW OF THE INVERSE.     GETROW      12
               JJ = 0                                                           GETROW      13
               DO 10 J = 1, N                                                   GETROW      14
                  AROW(J) = E(KROW+JJ)                                          GETROW      15
                  IF (ABS(AROW(J)).LE.EPS)  AROW(J) = 0.                        GETROW      16
                  JJ = JJ + N                                                   GETROW      17
         10       CONTINUE                                                      GETROW      18
               GO TO 140                                                        GETROW      19
         20 CONTINUE                                                            GETROW      20
      C        ORIGINAL DEPENDENT VARIABLE.                                     GETROW      21
      C        MULTIPLY ORIGINAL ROW BY THE INVERSE.                            GETROW      22
               KK = KROW - N                                                    GETROW      23
               L = IND(KK)                                                      GETROW      24
               JJ = 0                                                           GETROW      25
               DO 130 J = 1, N                                                  GETROW      26
                  AROW(J) = 0.                                                  GETROW      27
                  IF(KK .NE. M) GO TO 40                                        GETROW      28
      C          ELSE                                                           GETROW      29
                     DO 30 I = 1, N1                                            GETROW      30
                        AROW(J) = AROW(J) - OBJ(I)*E(I+JJ)                      GETROW      31
         30          CONTINUE                                                   GETROW      32
                     IF(N11 .GT. N) GO TO 120                                   GETROW      33
                     DO 34 I = N11, N                                           GETROW      34
                        AROW(J) = AROW(J) + OBJ(I)*E(I+JJ)                      GETROW      35
         34          CONTINUE                                                   GETROW      36
                     GO TO 120                                                  GETROW      37
      C          END                                                            GETROW      38
         40       IF(KK .GT. N1) GO TO 80                                       GETROW      39
      C          ELSE                                                           GETROW      40
                     DO 50 I = 1, N1                                            GETROW      41
                        AROW(J) = AROW(J) + Q(KK,I)*E(I+JJ)                     GETROW      42
         50          CONTINUE                                                   GETROW      43
                     IF(N11 .GT. N2) GO TO 65                                   GETROW      44
                     DO 60 I = N11, N2                                          GETROW      45
                        AROW(J) = AROW(J) + A1(IND(I),KK)*E(I+JJ)               GETROW      46
         60          CONTINUE                                                   GETROW      47
         65          IF(N21 .GT. N) GO TO 120                                   GETROW      48
                     DO 70 I = N21, N                                           GETROW      49
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    SUBROUTINE GETROW      73/74    OPT=1                           FTN 4.6+452        82/01/28. 14.34.48        PAGE      2

                             AROW(J) = AROW(J) + A2(IND(I),KK)*E(I+JJ)              GETROW      50
                  70       CONTINUE                                                 GETROW      51
  60                       GO TO 120                                                GETROW      52
            C           END                                                         GETROW      53
                  40     IF(KK .GT. N2) GO TO 100                                   GETROW      54
            C           ELSE                                                        GETROW      55
                           DO 90 I = 1, N1                                          GETROW      56
  65                         AROW(J) = AROW(J) + A1(L,I)*E(I+JJ)                    GETROW      57
                  90       CONTINUE                                                 GETROW      58
                           GO TO 120                                                GETROW      59
            C           END                                                         GETROW      60
                 100     DO 110 I = 1, N1                                           GETROW      61
  70                       AROW(J) = AROW(J) + A2(L,I)*E(I+JJ)                      GETROW      62
                 110     CONTINUE                                                   GETROW      63
                 120     CONTINUE                                                   GETROW      64
                         IF(ABS(AROW(J)) .LE. EPS) AROW(J) = 0.                     GETROW      65
                         JJ = JJ + N                                                GETROW      66
  75             130   CONTINUE                                                     GETROW      67
                 140 CONTINUE                                                       GETROW      68
                     RETURN                                                         GETROW      69
                     END                                                            GETROW      70


    SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
    4  GETROW         1            77

VARIABLES      SN  TYPE              RELOCATION
    0  AROW        REAL        ARRAY     F.P.       REFS       9      24      39      43      50      54      58
                                                      65      70      73  DEFINED       1      23      24      35
                                                      39      43      50      54      58      65      70      73
    0  A1          REAL        ARRAY     F.P.       REFS       9      54      65  DEFINED       1
    0  A2          REAL        ARRAY     F.P.       REFS       9      58      70  DEFINED       1
   20  BIGM        REAL                  XAXUP      REFS      14
    0  E           REAL        ARRAY     F.P.       REFS       9      23      39      43      50      54      58
                                                      65      70  DEFINED       1
    4  EPS         REAL                  XAXUP      REFS      12      24      73
  271  I           INTEGER                          REFS    2*39    2*43    2*50    2*54    2*58    2*65    2*70
                                                 DEFINED      38      42      49      53      57      64      69
    6  IERR        INTEGER               XAXUP      REFS      12
    0  IND         INTEGER     ARRAY     F.P.       REFS      10      32      54      58  DEFINED       1
    0  IOBJ        INTEGER               XAXUP      REFS      12
   11  IPIV        INTEGER               XAXUP      REFS      13
    5  ITER        INTEGER               XAXUP      REFS      12
    3  ITMAX       INTEGER               XAXUP      REFS      12
  266  J           INTEGER                          REFS      23    2*24      35    2*39    2*43    2*50    2*54
                                                    2*58    2*65    2*70    2*73  DEFINED      22      34
  265  JJ          INTEGER                          REFS      23      25      39      43      50      54      58
                                                      65      70      74  DEFINED      21      25      33      74
   12  JPIV        INTEGER               XAXUP      REFS      13
  267  KK          INTEGER                          REFS      32      36      47      50      54      58      62
                                                 DEFINED      31
    0  KROW        INTEGER               F.P.       REFS      19      23      31  DEFINED       1
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      SUBROUTINE GETROW      73/74    OPT=1                             FTN 4.6+452        82/01/28. 14.34.48        PAGE      3

VARIABLES       SN  TYPE             RELOCATION
  270  L            INTEGER                          REFS           65        70    DEFINED        32
    1  M            INTEGER                XXXOP     REFS           12        36
    2  N            INTEGER                XXXOP     REFS           12        19         22        25        31        34        41
                                                           42       56        57         74
   13  NEGV         LOGICAL                XXXOP     REFS           13        15
   21  NINVT        INTEGER                XXXOP     REFS           14
   10  NPM          INTEGER                XXXOP     REFS           13
    7  NP1          INTEGER                XXXOP     REFS           13
    0  NUD          INTEGER                 F.P.     REFS            9   DEFINED          1
   14  N1           INTEGER                XXXOP     REFS           13        38         47        49        64        69
    0  N1D          INTEGER                 F.P.     REFS            9   DEFINED          1
   15  N11          INTEGER                XXXOP     REFS           13        41         42        52        53
   16  N2           INTEGER                XXXOP     REFS           13        52         53        62
    0  N2D          INTEGER                 F.P.     REFS            9   DEFINED          1
   17  N21          INTEGER                XXXOP     REFS           13        56         57
    0  OBJ          REAL          ARRAY     F.P.     REFS            9        39         43   DEFINED          1
   22  PHASE1       LOGICAL                XXXOP     REFS           14        15
    0  Q            REAL          ARRAY     F.P.     REFS            9        50    DEFINED         1

INLINE FUNCTIONS    TYPE     ARGS         DEF LINE  REFERENCES
      ABS.         REAL       1  INTRIN                   24          73

STATEMENT LABELS                   DEF LINE    REFERENCES
    0  10                             26         22
   52  20                             28         19
    0  30                             40         38
    0  34                             44         42
  117  40                             47         46
    0  50                             51         49
    0  60                             55         53
  162  65                             56         52
    0  70                             59         57
  206  80                             62         47
    0  90                             66         64
  230  100                            69         62
    0  110                            71         69
  246  120                            72         41         45         56         60         67
    0  130                            75         34
  257  140                            76         27

LOUPS  LABEL      INDEX      FROM-TO     LENGTH     PROPERTIES
   40  10           J         22 26         7B       INSTACK
   60  130        * J         34 75       176B                      NOT INNER
   73  30           I         38 40         3B       INSTACK
  111  34           I         42 44         3B       INSTACK
  133  50           I         49 51         3B       INSTACK
  154  60           I         53 55         4B       INSTACK
  177  70           I         57 59         4B       INSTACK
  222  90           I         64 66         3B       INSTACK
  241  110          I         69 71         3B       INSTACK

COMMON BLOCKS   LENGTH
       XXXOP        19

STATISTICS
  PROGRAM LENGTH                     342B       226
  CM LABELED COMMON LENGTH            23B        19
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      SUBROUTINE GETROW      73/74   OPT=1                          FTN 4.6+452        82/01/28. 14.34.48       PAGE     4

   STATISTICS
        52000B CM USED
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SUBROUTINE PSOL       73/74    OPT=1                    FTN 4.6+452        82/01/28. 14.34.48       PAGE      1

                SUBROUTINE PSOL(Q,A1,A2,OBJ,IND,K,X,E,NQD,N1D,N2D)                  PSOL          2
          C                                                                         LINE          2
          C-------------------------------------------------------------------------LINE          3
          C                                                                         LINE          4
          C     PRIMAL SOLUTION                                                     PSOL          4
          C                                                                         LINE          2
          C-------------------------------------------------------------------------LINE          3
          C                                                                         LINE          4
                DIMENSION Q(NQD,1), A1(N1D,1), A2(N2D,1), K(1), X(1), E(1)         PSOL          6
                DIMENSION OBJ(1), IND(1)                                            PSOL          7
          C                                                                         PSOL          8
                COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                  /XXXQP/       2
                COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NFGV, N1, N11, N2, N21        /XXXQP/       3
                COMMON /XXXQP/ BIGM, NINVT, PHASE1                                 /XXXQP/       4
                LOGICAL NEGV, PHASE1                                               /XXXQP/       5
          C                                                                         LINE          2
          C-------------------------------------------------------------------------LINE          3
          C                                                                         LINE          4
                DO 30 I = NP1, NPM                                                 PSOL         11
                   KI = K(I)                                                        PSOL         12
                   IF (KI.GT.N)  GO TO 20                                           PSOL         13
                      X(KI) = 0.                                                    PSOL         14
                      JJ = 0                                                        PSOL         15
                      DO 10 J = 1, N                                                PSOL         16
                         X(KI) = X(KI) + E(KI+JJ) * X(K(J))                         PSOL         17
                         JJ = JJ + N                                                PSOL         18
             10          CONTINUE                                                   PSOL         19
             20    CONTINUE                                                         PSOL         20
             30    CONTINUE                                                         PSOL         21
                DO 130 I = NP1, NPM                                                PSOL         22
                   KI = K(I)                                                        PSOL         23
                   IF(KI .LE. N) GO TO 130                                          PSOL         24
                      X(KI) = 0.                                                    PSOL         25
                      KK = KI - N                                                   PSOL         26
                      L = IND(KK)                                                   PSOL         27
                      IF(KK .GT. N1) GO TO 70                                       PSOL         28
          C          ELSE                                                           PSOL         29
                         DO 40 J = 1, N1                                            PSOL         30
                            X(KI) = X(KI) + Q(KK,J)*X(J)                            PSOL         31
             40          CONTINUE                                                   PSOL         32
                         IF(N11 .GT. N2) GO TO 55                                   PSOL         33
                         DO 50 J = N11, N2                                          PSOL         34
                            X(KI) = X(KI) + A1(IND(J),KK)*X(J)                      PSOL         35
             50          CONTINUE                                                   PSOL         36
             55          IF(N21 .GT. N) GO TO 130                                   PSOL         37
                         DO 60 J = N21, N                                           PSOL         38
                            X(KI) = X(KI) + A2(IND(J),KK)*X(J)                      PSOL         39
             60          CONTINUE                                                   PSOL         40
                         GO TO 130                                                  PSOL         41
          C          END                                                            PSOL         42
             70       IF(KK .GT. N2) GO TO 90                                       PSOL         43
          C          ELSE                                                           PSOL         44
                         DO 80 J = 1, N1                                            PSOL         45
                            X(KI) = X(KI) + A1(L,J)*X(J)                            PSOL         46
             80          CONTINUE                                                   PSOL         47
                         GO TO 130                                                  PSOL         48
          C          END                                                            PSOL         49
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     SUBROUTINE PSOL       73/74    OPT=1                       FTN 4.6+452        82/01/28. 14.34.48       PAGE     2

                90        IF(KK .GT. N) GO TO 110                                 PSOL       50
              C          ELSE                                                     PSOL       51
                             DO 100 J = 1, N1                                     PSOL       52
                                X(KI) = X(KI) + A2(L,J)*X(J)                      PSOL       53
               100           CONTINUE                                             PSOL       54
                             GO TO 130                                            PSOL       55
              C          END                                                      PSOL       56
               110        DO 120 J = 1, N1                                        PSOL       57
                             X(KI) = X(KI) - OBJ(J)*X(J)                          PSOL       58
               120        CONTINUE                                                PSOL       59
                          IF(N11 .GT. N) GO TO 130                                PSOL       60
                          DO 124 J = N11, N
                             X(KI) = X(KI) + OBJ(J)*X(J)                          PSOL       61
               124        CONTINUE                                                PSOL       62
              C       END                                                         PSOL       63
               130 CONTINUE                                                       PSOL       64
                   RETURN                                                         PSOL       65
                   END                                                            PSOL       66
                                                                                  PSOL       67

     SYMBOLIC REFERENCE MAP (R=2)

ENTRY POINTS      DEF LINE      REFERENCES
    4 PSOL            1           74

VARIABLES       SN  TYPE            RELOCATION
    0  A1           REAL        ARRAY      F.P.         REFS          9          43        54    DEFINED          1
    0  A2           REAL        ARRAY      F.P.         REFS          9          47        61    DEFINED          1
   20  BIGM         REAL                   XXXOP        REFS         14
    0  E            REAL        ARRAY      F.P.         REFS          9          25   DEFINED          1
    4  EPS          REAL                   XXXOP        REFS         12
  254  I            INTEGER                             REFS         20          31   DEFINED         19          30
    6  IERR         INTEGER                XXXOP        REFS         12
    0  IND          INTEGER     ARRAY      F.P.         REFS         10          35        43         47    DEFINED          1
    0  IOBJ         INTEGER                XXXOP        REFS         12
   11  IPIV         INTEGER                XXXOP        REFS         13
    5  ITER         INTEGER                XXXOP        REFS         12
    3  ITMAX        INTEGER                XXXOP        REFS         12
  257  J            INTEGER                             REFS         25        2*39      2*43       2*47       2*54       2*61       2*66
                                                        2*70    DEFINED          24        38         42         46         53         60
                                                          65         69
  256  JJ           INTEGER                             REFS         25          26   DEFINED         23          26
   12  JPIV         INTEGER                XXXOP        REFS         13
    0  K            INTEGER     ARRAY      F.P.         REFS          9          20        25         31    DEFINED          1
  255  KI           INTEGER                             REFS         21          22      3*25         32         33         34       2*39
                                                        2*43       2*47        2*54      2*61       2*66       2*70
                                                      DEFINED        20          31
  260  KK           INTEGER                             REFS         35          36        39         43         47         51         58
                                                      DEFINED        34
  261  L            INTEGER                             REFS         54          61   DEFINED         35
    1  M            INTEGER                XXXOP        REFS         12
    2  N            INTEGER                XXXOP        REFS         12          21        24         26         32         34         45
                                                          46         58          60        69
   13  NEGV         LOGICAL                XXXOP        REFS         13          15
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        SUBROUTINE PSOL        73/74    OPT=1                         FTN 4.6+452         82/01/28. 14.34.48        PAGE      3

VARIABLES      SN  TYPE            RELOCATION
   21  NINVT      INTEGER                XXXOP      REFS        14
   10  NPM        INTEGER                XXXOP      REFS        13        19        30
    7  NP1        INTEGER                XXXOP      REFS        13        19        30
    0  NOD        INTEGER                 F.P.      REFS         9   DEFINED         1
   14  N1         INTEGER                XXXOP      REFS        13        36        38        53        60        65
    0  NID        INTEGER                 F.P.      REFS         9   DEFINED         1
   15  N11        INTEGER                XXXOP      REFS        13        41        42        68        69
   16  N2         INTEGER                XXXOP      REFS        13        41        42        51
    0  N2D        INTEGER                 F.P.      REFS         9   DEFINED         1
   17  N21        INTEGER                XXXOP      REFS        13        45        46
    0  OBJ        REAL        ARRAY       F.P.      REFS        10        66        70   DEFINED         1
   22  PHASE1     LOGICAL                XXXOP      REFS        14        15
    0  U          REAL        ARRAY       F.P.      REFS         9        39   DEFINED         1
    0  X          REAL        ARRAY       F.P.      REFS         9      2*25      2*39      2*43      2*47      2*54      2*61
                                                      2*66      2*70   DEFINED         1        22        25        33        39
                                                        43        47        54        61        66        70

STATEMENT LABELS               DEF LINE    REFERENCES
    0  10                          27        24
   52  20 *                        28        21
    0  30                          29        19
    0  40                          40        38
    0  50                          44        42
  126  55                          45        41
    0  60                          48        46
  151  70                          51        36
    0  80                          55        53
  172  90                          58        51
    0  100                         62        60
  213  110                         65        58
    0  120                         67        65
    0  124                         71        69
  243  130                         73        30        32        45        49        56        63        68

LOOPS  LABEL     INDEX       FROM-TO      LENGTH     PROPERTIES
   26  30       * I          19 29        27B                  NOT INNER
   43  10         J          24 27         5B         INSTACK
   57  130      * I          30 73       167B                  NOT INNER
  100  40         J          38 40         3B         INSTACK
  120  50         J          42 44         4B         INSTACK
  142  60         J          46 48         4B         INSTACK
  164  80         J          53 55         3B         INSTACK
  205  100        J          60 62         3B         INSTACK
  221  120        J          65 67         3B         INSTACK
  236  124        J          69 71         3B         INSTACK

COMMON BLOCKS    LENGTH
       XXXOP          19

STATISTICS
  PROGRAM LENGTH                     340B        224
  CM LABELED COMMON LENGTH            23B         19
        52000B CM USED
```
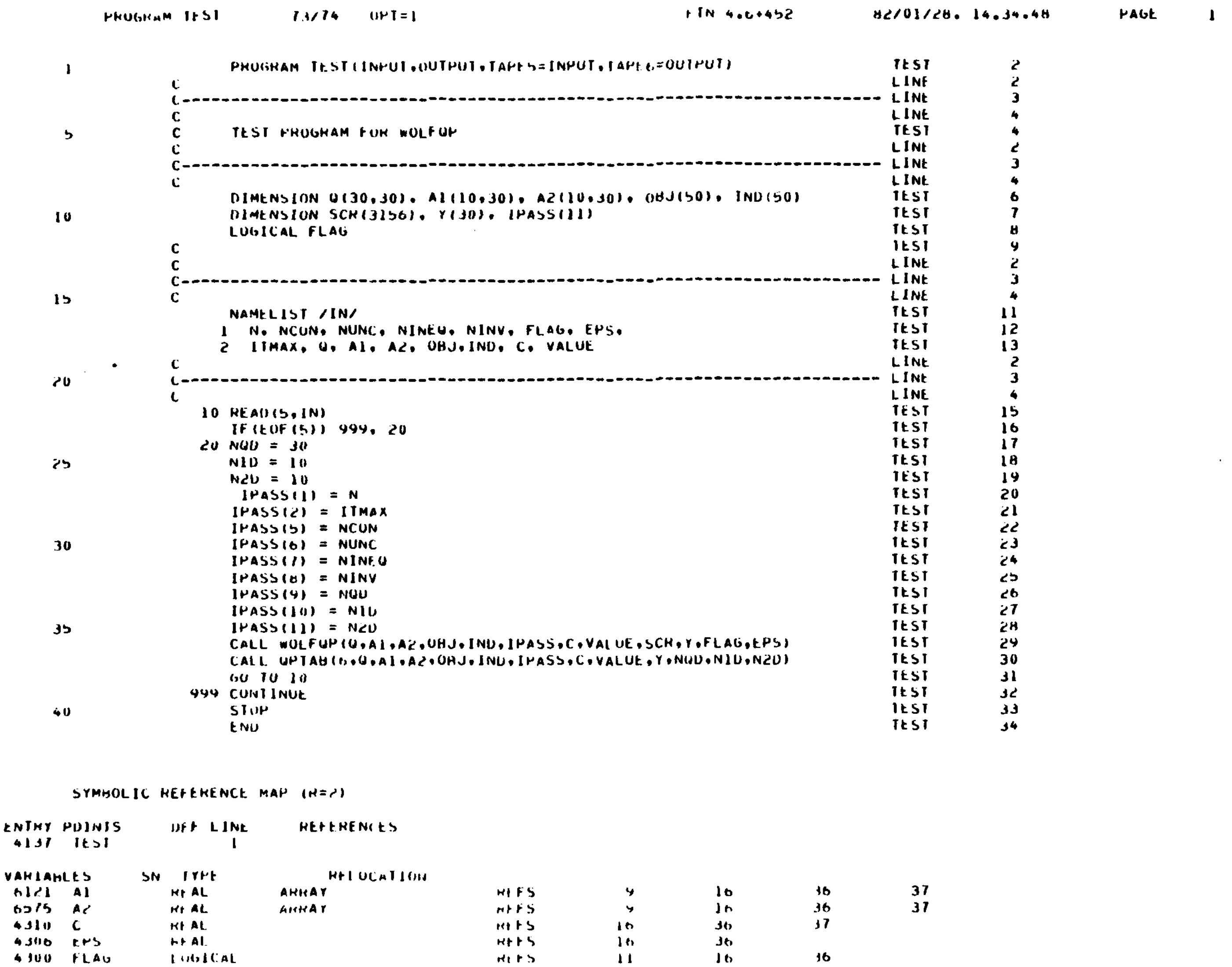

```
SUBROUTINE SETINV      73/74    OPT=1                 FTN 4.6+452        82/01/28. 14.34.48        PAGE      1

            SUBROUTINE SETINV(Q,A1,A2,OBJ,IND,K,E,AROW,NQD,N1D,N2D)          SETINV      2
      C                                                                       LINE        2
      C---------------------------------------------------------------------- LINE        3
      C                                                                       LINE        4
      C     INITIAL INVERSE                                                   SETINV      4
      C                                                                       LINE        2
      C---------------------------------------------------------------------- LINE        3
      C                                                                       LINE        4
            DIMENSION K(1), E(1), AROW(1)                                     SETINV      6
      C                                                                       SETINV      7
            COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR                 /XXXQP/     2
            COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21       /XXXQP/     3
            COMMON /XXXQP/ HIGM, NINVT, PHASE1                                /XXXQP/     4
            LOGICAL NEGV, PHASE1                                              /XXXQP/     5
      C                                                                       LINE        2
      C---------------------------------------------------------------------- LINE        3
      C                                                                       LINE        4
      C     SET E TO THE IDENTITY                                             SETINV     10
            JJ = 0                                                            SETINV     11
            DO 20 J = 1, N                                                    SETINV     12
               DO 10 I = 1, N                                                 SETINV     13
                  E(I+JJ) = 0.                                                SETINV     14
         10       CONTINUE                                                    SETINV     15
               E(J+JJ) = 1.                                                   SETINV     16
               JJ = JJ + N                                                    SETINV     17
         20    CONTINUE                                                       SETINV     18
      C     GENERATE INITIAL INVERSE                                          SETINV     19
            DO 30 J = 1, N                                                    SETINV     20
               K(J) = -K(J)                                                   SETINV     21
         30    CONTINUE                                                       SETINV     22
            DO 90 JJ = 1, N                                                   SETINV     23
               DO 40 J = 1, N                                                 SETINV     24
                  IF (K(J).LT.0)  GO TO 50                                    SETINV     25
         40       CONTINUE                                                    SETINV     26
         50    CONTINUE                                                       SETINV     27
               KROW = -K(J)                                                   SETINV     28
               CALL GETROW(Q,A1,A2,OBJ,IND,E,KROW,AROW,NQD,N1D,N2D)           SETINV     29
               ROWMAX = 0.                                                    SETINV     30
               DO 70 L = 1, N                                                 SETINV     31
                  TEST = ABS(AROW(L))                                         SETINV     32
                  IF (K(L).GT.0 .OR. TEST.LT.ROWMAX)  GO TO 60                SETINV     33
                     ROWMAX = TEST                                            SETINV     34
                     JPIV = L                                                 SETINV     35
         60       CONTINUE                                                    SETINV     36
         70       CONTINUE                                                    SETINV     37
               IF (ROWMAX.GT.0.)  GO TO 80                                    SETINV     38
                  IERR = 3                                                    SETINV     39
                  RETURN                                                      SETINV     40
         80    CONTINUE                                                       SETINV     41
               K(J) = K(JPIV)                                                 SETINV     42
               K(JPIV) = KROW                                                 SETINV     43
               CALL NEWINV(E,AROW)                                            SETINV     44
         90    CONTINUE                                                       SETINV     45
            ITER = 0                                                          SETINV     46
            RETURN                                                            SETINV     47
            END                                                               SETINV     48
      SUBROUTINE SETINV      74/74    OPT=1                    FTN 4.6+452       82/01/28. 14.34.48        PAGE     2

      SYMBOLIC REFERENCE MAP (R=2)

 ENTRY POINTS      DEF LINE      REFERENCES
     4  SETINV         1           48        55

 VARIABLES       SN   TYPE              RELOCATION
      0  AROW         REAL          ARRAY     F.P.          REFS        9        37       40       52    DEFINED       1
      0  A1           REAL                    F.P.          REFS       37   DEFINED        1
      0  A2           REAL                    F.P.          REFS       37   DEFINED        1
     20  BIGM         REAL                    XXXUP         REFS       13
      0  E            REAL          ARRAY     F.P.          REFS        9        37       52  DEFINED        1        22       24
      4  EPS          REAL                    XXXUP         REFS       11
    164  I            INTEGER                               REFS       22   DEFINED       21
      6  IERR         INTEGER                 XXXUP         REFS       11   DEFINED       47
      0  IND          INTEGER                 F.P.          REFS       37   DEFINED        1
      0  IOBJ         INTEGER                 XXXUP         REFS       11
     11  IPIV         INTEGER                 XXXUP         REFS       12
      5  ITER         INTEGER                 XXXUP         REFS       11   DEFINED       54
      3  ITMAX        INTEGER                 XXXUP         REFS       11
    163  J            INTEGER                               REFS       24      2*29       33       36       50
                                                         DEFINED       20        28       32
    162  JJ           INTEGER                               REFS       22        24       25  DEFINED       19        25       31
     12  JPIV         INTEGER                 XXXUP         REFS       12        50       51  DEFINED       43
      0  K            INTEGER       ARRAY     F.P.          REFS        9        29       33       36       41        50
                                                         DEFINED        1        29       50       51
    165  KROW         INTEGER                               REFS       37        51  DEFINED       36
    167  L            INTEGER                               REFS       40        41       43  DEFINED       39
      1  M            INTEGER                 XXXUP         REFS       11
      2  N            INTEGER                 XXXUP         REFS       11        20       21       25       28        31       32
                                                             39
     13  NEGV         LOGICAL                 XXXUP         REFS       12        14
     21  NINVT        INTEGER                 XXXUP         REFS       13
     10  NPM          INTEGER                 XXXUP         REFS       12
      7  NP1          INTEGER                 XXXUP         REFS       12
      0  NOD          INTEGER                 F.P.          REFS       37   DEFINED        1
     14  N1           INTEGER                 XXXUP         REFS       12
      0  N1D          INTEGER                 F.P.          REFS       37   DEFINED        1
     15  N11          INTEGER                 XXXUP         REFS       12
     16  N2           INTEGER                 XXXUP         REFS       12
      0  N2D          INTEGER                 F.P.          REFS       37   DEFINED        1
     17  N21          INTEGER                 XXXUP         REFS       12
      0  OBJ          REAL                    F.P.          REFS       37   DEFINED        1
     22  PHASE1       LOGICAL                 XXXUP         REFS       13        14
      0  W            REAL                    F.P.          REFS       37   DEFINED        1
    166  ROWMAX       REAL                                  REFS       41        46  DEFINED       38       42
    170  TEST         REAL                                  REFS       41        42  DEFINED       40

 EXTERNALS              TYPE    ARGS         REFERENCES
        GETROW                   11              37
        NEWINV                    2              52

 INLINE FUNCTIONS    TYPE    ARGS            DEF LINE   REFERENCES
        ABS          REAL       1   INTRIN                    40
     SUBROUTINE SETINV      73/74    OPT=1                    FTN 4.6+452        82/01/28. 14.34.48       PAGE      3

STATEMENT LABELS                   DEF LINE   REFERENCES
    0  10                              23         21
    0  20                              26         20
    0  30                              30         28
    0  40                              34         32
   65  50                              35         33
  117  60                              44         41
    0  70                              45         39
  126  80                              49         46
    0  90                              53         31

LOOPS  LABEL     INDEX      FROM-TO     LENGTH     PROPERTIES
   22  20      * J           20 26        20B                NOT INNER
   31  10        I           21 23         2B       INSTACK
   50  30        J           28 30         2B       INSTACK
   55  90      * JJ          31 53        63B                EXT REFS  NOT INNER
   57  40      * J           32 34         5B       INSTACK    EXITS
  111  70        L           39 45         7B       INSTACK

COMMON BLOCKS   LENGTH
       XXXUP        19

STATISTICS
  PROGRAM LENGTH                  230B      152
  CM LABELED COMMON LENGTH         23B       19
          52000B CM USED
     SUBROUTINE NEWINV     73/74*  OPT=1                               FTN 4.6+452        82/01/28. 14.34.48        PAGE     1

 1                SUBROUTINE NEWINV(E,AROW)                                       NEWINV      2
            C                                                                     LINE        2
            C-------------------------------------------------------------------- LINE        3
            C                                                                     LINE        4
 5          C     INVERSE UPDATE BY COLUMN OPERATIONS                             NEWINV      4
            C                                                                     LINE        2
            C-------------------------------------------------------------------- LINE        3
            C                                                                     LINE        4
                  DIMENSION E(1), AROW(1)                                         NEWINV      6
 10         C                                                                     NEWINV      7
                  COMMON /XXXQP/ IOBJ, M, N, ITMAX, EPS, ITER, IERR               /XXXQP/     2
                  COMMON /XXXQP/ NP1, NPM, IPIV, JPIV, NEGV, N1, N11, N2, N21     /XXXQP/     3
                  COMMON /XXXQP/ BIGM, NINVT, PHASE1                              /XXXQP/     4
                  LOGICAL NEGV, PHASE1                                            /XXXQP/     5
 15         C                                                                     LINE        2
            C-------------------------------------------------------------------- LINE        3
            C                                                                     LINE        4
                  JJPIV = (JPIV-1)*N                                              NEWINV     10
                  DO 20 I = 1, N                                                  NEWINV     11
 20                  EPIV = E(I+JJPIV)/AROW(JPIV)                                 NEWINV     12
                     IF(ABS(EPIV)-EPS) 15, 20, 15                                 NEWINV     13
               15    JJ = 0                                                       NEWINV     14
                     DO 10 J = 1, N                                               NEWINV     15
                        E(I+JJ) = E(I+JJ) - EPIV*AROW(J)                          NEWINV     16
 25                     JJ = JJ + N                                               NEWINV     17
               10       CONTINUE                                                  NEWINV     18
                     E(I+JJPIV) = EPIV                                            NEWINV     19
               20    CONTINUE                                                     NEWINV     20
                  RETURN                                                          NEWINV     21
 30               END                                                             NEWINV     22


     SYMBOLIC REFERENCE MAP (R=2)

 ENTRY POINTS      DEF LINE      REFERENCES
     4  NEWINV          1          29

 VARIABLES       SN  TYPE             RELOCATION
     0  AROW        REAL         ARRAY    F.P.        REFS        9       20       24    DEFINED       1
    20  BIGM        REAL                  XXXQP       REFS       13
     0  E           REAL         ARRAY    F.P.        REFS        9       20       24    DEFINED       1        24        27
    60  EPIV        REAL                              REFS       21       24       27    DEFINED      20
     4  EPS         REAL                  XXXQP       REFS       11       21
    57  I           INTEGER                           REFS       20     2*24       27    DEFINED      19
     6  IERR        INTEGER               XXXQP       REFS       11
     0  IOBJ        INTEGER               XXXQP       REFS       11
    11  IPIV        INTEGER               XXXQP       REFS       12
     5  ITER        INTEGER               XXXQP       REFS       11
     3  ITMAX       INTEGER               XXXQP       REFS       11
    62  J           INTEGER                           REFS       24   DEFINED       23
    61  JJ          INTEGER                           REFS     2*24       25  DEFINED       22        25
    56  JJPIV       INTEGER                           REFS       20       27  DEFINED       18
    12  JPIV        INTEGER               XXXQP       REFS       12       18       20
     1  M           INTEGER               XXXQP       REFS       11
     SUBROUTINE NEWINV     73/74    OPT=1                              FTN 4.6+452      82/01/28. 14.34.48      PAGE    2

VARIABLES     SN  TYPE            RELOCATION
    2  N          INTEGER             XXXUP      REFS      11      18      19      23      25
   13  NEGV       LOGICAL             XXXUP      REFS      12      14
   21  NINVT      INTEGER             XXXUP      REFS      13
   10  NPM        INTEGER             XXXUP      REFS      12
    7  NP1        INTEGER             XXXUP      REFS      12
   14  N1         INTEGER             XXXUP      REFS      12
   15  N11        INTEGER             XXXUP      REFS      12
   16  N2         INTEGER             XXXUP      REFS      12
   17  N21        INTEGER             XXXUP      REFS      12
   22  PHASE1     LOGICAL             XXXUP      REFS      13      14

INLINE FUNCTIONS   TYPE   ARGS        DEF LINE  REFERENCES
       ABS         REAL     1  INTRIN                21

STATEMENT LABELS                 DEF LINE   REFERENCES
    0  10                           26         23
    0  15           INACTIVE        22         2*21
   52  20                           28         19        21

LOOPS  LABEL    INDEX     FROM-TO     LENGTH     PROPERTIES
   23  20      * I         19 28        32B                NOT INNER
   41  10        J         23 26         5B       INSTACK

COMMON BLOCKS    LENGTH
       XXXUP        19

STATISTICS
  PROGRAM LENGTH                  72B       58
  CM LABELLED COMMON LENGTH       23B       19
         52000B CM USED
```

```
SUBROUTINE QPTAB       73/74    OPT=1                          FTN 4.6+452        82/01/28. 14.34.48        PAGE     1

                SUBROUTINE QPTAB(IOUT,Q,A1,A2,P,IND,IPASS,C,VALUE,Y,                   QPTAB         2
               *                 NQD,N1D,N2D)                                          QPTAB         3
              C                                                                        LINE          2
              C----------------------------------------------------------------------- LINE          3
              C                                                                        LINE          4
              C      OUTPUT QUADRATIC PROGRAM TABLEAU                                  QPTAB         5
              C      WOLFQP MUST HAVE PREVIOUSLY BEEN CALLED TO                        QPTAB         6
              C      RECEIVE THE COMPLETE OUTPUT                                       QPTAB         7
              C                                                                        QPTAB         8
                DIMENSION Q(NQD,1), A1(N1D,1), A2(N2D,1), IPASS(11)                    QPTAB         9
                DIMENSION P(1), IND(1), Y(1)                                           QPTAB        10
              C                                                                        QPTAB        11
              C                                                                        QPTAB        12
                NAMELIST /OUT/                                                         QPTAB        13
               1  N, NCON, NUNC, NINEQ, NEQ, NINV,                                     QPTAB        14
               2  ITMAX, ITER, IERR, C, VALUE                                          QPTAB        15
              C                                                                        QPTAB        16
              C                                                                        QPTAB        17
              1 FORMAT(1H0,9X,1HQ,/)                                                   QPTAB        18
              2 FORMAT(1H0,9X,1HP,/)                                                   QPTAB        19
              3 FORMAT(1X,10(2X,E10.4))                                                QPTAB        20
              4 FORMAT(1H0,4X,4HA1*X,9X,2HB1,15X,2HA1,/)                               QPTAB        21
              5 FORMAT(1H0,4X,4HA2*X,9X,2HB2,15X,2HA2,/)                               QPTAB        22
              6 FORMAT(1H0)                                                            QPTAB        23
              7 FORMAT(1H ,24X,8(2X,E10.4))                                            QPTAB        24
              8 FORMAT(1H0,9X,1HX,/)                                                   QPTAB        25
              C                                                                        LINE          2
              C----------------------------------------------------------------------- LINE          3
              C                                                                        LINE          4
                N = IPASS(1)                                                           QPTAB        27
                NCON = IPASS(5)                                                        QPTAB        28
                NUNC = IPASS(6)                                                        QPTAB        29
                NINEQ = IPASS(7)                                                       QPTAB        30
                NEQ = N - (NCON + NUNC + NINEQ)                                        QPTAB        31
                NINV = IPASS(8)                                                        QPTAB        32
                ITMAX = IPASS(2)                                                       QPTAB        33
                ITER = IPASS(3)                                                        QPTAB        34
                IERR = IPASS(4)                                                        QPTAB        35
                NN = NCON + NUNC                                                       QPTAB        36
                NN1 = NCON + NUNC + NINEQ                                              QPTAB        37
                WRITE(IOUT,OUT)                                                        QPTAB        38
                WRITE(IOUT,1)                                                          QPTAB        39
                DO 110 I = 1, NN, 10                                                   QPTAB        40
                   N1 = MIN0(I+9,NN)                                                   QPTAB        41
                   DO 100 J = 1, NN                                                    QPTAB        42
                      WRITE(IOUT,3)(Q(J,L),L=I,N1)                                     QPTAB        43
            100    CONTINUE                                                            QPTAB        44
                   WRITE(IOUT,6)                                                       QPTAB        45
            110 CONTINUE                                                               QPTAB        46
                WRITE(IOUT,2)                                                          QPTAB        47
                DO 120 I = 1, NN, 10                                                   QPTAB        48
                   N1 = MIN0(I+9,NN)                                                   QPTAB        49
                   WRITE(IOUT,3)(P(L),L=I,N1)                                          QPTAB        50
            120 CONTINUE                                                               QPTAB        51
                IF(NINEQ .LE. 0) GO TO 160                                             QPTAB        52
                WRITE(IOUT,4)                                                          QPTAB        53
                DO 150 I = 1, NN, 8                                                    QPTAB        54
```

```
      SUBROUTINE OPTAB       73/74    OPT=1                     FTN 4.6+452        82/01/28. 14.34.48       PAGE     2

                    NI = MIN0(I+7,NN)                                                    OPTAB       55
                    DO 140 L = 1, NINEQ                                                  OPTAB       56
                       LL = IND(L,NN)                                                    OPTAB       57
                       IF(I .GT. 1) GO TO 135                                            OPTAB       58
              C       ELSE                                                               OPTAB       59
                          SUM = 0.                                                       OPTAB       60
                          DO 130 K = 1, NN                                               OPTAB       61
                             SUM = SUM + A1(LL,K)*Y(K)                                   OPTAB       62
                130       CONTINUE                                                       OPTAB       63
                          WRITE(IOUT,3) SUM, P(NN+L), (A1(LL,K),K=I,NI)                  OPTAB       64
                          GO TO 140                                                      OPTAB       65
              C       END                                                                OPTAB       66
                135    WRITE(IOUT,7)(A1(LL,K),K=I,NI)                                    OPTAB       67
                140 CONTINUE                                                             OPTAB       68
                    WRITE(IOUT,6)                                                        OPTAB       69
                150 CONTINUE                                                             OPTAB       70
                160 IF(NEQ .LE. 0) GO TO 200                                             OPTAB       71
                    WRITE(IOUT,5)                                                        OPTAB       72
                    DO 190 I = 1, NN, 8                                                  OPTAB       73
                       NI = MIN0(I+7,NN)                                                 OPTAB       74
                       DO 180 L = 1, NEQ                                                 OPTAB       75
                          LL = IND(L,NN1)                                                OPTAB       76
                          IF(I .GT. 1) GO TO 175                                         OPTAB       77
              C       ELSE                                                               OPTAB       78
                          SUM = 0.                                                       OPTAB       79
                          DO 170 K = 1, NN                                               OPTAB       80
                             SUM = SUM + A2(LL,K)*Y(K)                                   OPTAB       81
                170       CONTINUE                                                       OPTAB       82
                          WRITE(IOUT,3) SUM, P(NN1+L), (A2(LL,K),K=I,NI)                 OPTAB       83
                          GO TO 180                                                      OPTAB       84
              C       END                                                                OPTAB       85
                175    WRITE(IOUT,7)(A2(LL,K),K=I,NI)                                    OPTAB       86
                180    CONTINUE                                                          OPTAB       87
                       WRITE(IOUT,6)                                                     OPTAB       88
                190 CONTINUE                                                             OPTAB       89
                200 WRITE(IOUT,8)                                                        OPTAB       90
                    DO 210 I = 1, NN, 10                                                 OPTAB       91
                       NI = MIN0(I+9,NN)                                                 OPTAB       92
                       WRITE(IOUT,3)(Y(L),L=I,NI)                                        OPTAB       93
                210 CONTINUE                                                             OPTAB       94
                    RETURN                                                               OPTAB       95
                    END                                                                  OPTAB       96


      SYMBOLIC REFERENCE MAP (R=2)

  ENTRY POINTS      DEF LINE     REFERENCES
     4  OPTAB           1           98

  VARIABLES      SN   TYPE           RELOCATION
      0  A1           REAL         ARRAY      F.P.       REFS        10          65         67         70     DEFINED        1
      0  A2           REAL         ARRAY      F.P.       REFS        10          84         86         89     DEFINED        1
      0  C            REAL                    F.P.       REFS        14     DEFINED          1
    572  I            INTEGER                            REFS        44          46         52         53         58        61        67
```

```
       SUBROUTINE OPTAB       73/74    OPT=1                            FTN 4.6+452       82/01/28. 14.34.46       PAGE     3

VARIABLES      SN  TYPE              RELOCATION
                                                         70        77        80        86        89        95        96
                                                    DEFINED        43        51        57        76        94
   567  IERR        INTEGER                            REFS        14   DEFINED        38
     0  IND         INTEGER     ARRAY       F.P.       REFS        11        60        79   DEFINED         1
     0  IOUT        INTEGER                 F.P.    DEFINED         1  I/O REFS        41        42        46        48        50
                                                         53        56        67        70        72        75        86        89
                                                         91        93        96
     0  IPASS       INTEGER     ARRAY       F.P.       REFS        10        30        31        32        33        35        36
                                                         37        38   DEFINED         1
   566  ITER        INTEGER                            REFS        14   DEFINED        37
   565  ITMAX       INTEGER                            REFS        14   DEFINED        36
   574  J           INTEGER                            REFS        46   DEFINED        45
   600  K           INTEGER                            REFS      2*65        67        70      2*84        86        89
                                                    DEFINED        64        67        70        83        86        89
   575  L           INTEGER                            REFS        46        53        60        67        79        86        96
                                                    DEFINED        46        53        59        78        96
   576  LL          INTEGER                            REFS        65        67        70        84        86        89
                                                    DEFINED        60        79
   557  N           INTEGER                            REFS        14        34   DEFINED        30
   560  NCON        INTEGER                            REFS        14        34        39        40   DEFINED        31
   563  NEQ         INTEGER                            REFS        14        74        78   DEFINED        34
   562  NINEQ       INTEGER                            REFS        14        34        40        55        59
                                                    DEFINED        33
   564  NINV        INTEGER                            REFS        14   DEFINED        35
   570  NN          INTEGER                            REFS        43        44        45        51        52        57        58
                                                         60        64        67        76        77        83        94        95
                                                    DEFINED        39
   571  NN1         INTEGER                            REFS        79        86   DEFINED        40
     0  NOD         INTEGER                 F.P.       REFS        10   DEFINED         1
   561  NUNC        INTEGER                            REFS        14        34        39        40   DEFINED        32
   573  N1          INTEGER                            REFS        46        53        67        70        86        89        96
                                                    DEFINED        44        52        58        77        95
     0  N1D         INTEGER                 F.P.       REFS        10   DEFINED         1
     0  N2D         INTEGER                 F.P.       REFS        10   DEFINED         1
     0  P           REAL        ARRAY       F.P.       REFS        11        53        67        86   DEFINED         1
     0  Q           REAL        ARRAY       F.P.       REFS        10        46   DEFINED         1
   577  SUM         REAL                               REFS        65        67        84        86   DEFINED        63        65
                                                         82        84
     0  VALUE       REAL                    F.P.       REFS        14   DEFINED         1
     0  Y           REAL        ARRAY       F.P.       REFS        11        65        84        96   DEFINED         1
        VARIABLES USED AS FILE NAMES, SEE ABOVE

INLINE FUNCTIONS    TYPE     ARGS          DEF LINE   REFERENCES
        MINO        INTEGER     0  INTRIN                  44        52        58        77        95

NAMELISTS           DEF LINE        REFERENCES
        OUT             14             41

STATEMENT LABELS                    DEF LINE    REFERENCES
   421  1         FMT                   19          42
   424  2         FMT                   20          50
   427  3         FMT                   21          46        53        67        86        96
   432  4         FMT                   22          56
   436  5         FMT                   23          75
   442  6         FMT                   24          48        72        91
   444  7         FMT                   25          70        89
   447  8         FMT                   26          93
```

```
        SUBROUTINE OPTAB        73/74    OPT=1                         FTN 4.6+452        82/01/28. 14.34.48        PAGE     4

STATEMENT LABELS                DEF LINE    REFERENCES
    0  100                          47          45
    0  110                          49          43
    0  120                          54          51
    0  130                          66          64
  204  135                          70          61
  223  140                          71          59          68
    0  150                          73          57
  234  160                          74          55
    0  170                          85          83
  314  175                          89          80
  333  180                          90          78          87
    0  190                          92          76
  344  200                          93          74
    0  210                          97          94

LOUPS  LABEL     INDEX       FROM-TO      LENGTH     PROPERTIES
   47  110     * I          43 49          34B               EXT REFS  NOT INNER
   54  100     * J          45 47          22B               EXT REFS  NOT INNER
   60          * L          46 46          11B               EXT REFS
  106  120     * I          51 54          15B               EXT REFS
  130  150     * I          57 73         103B               EXT REFS  NOT INNER
  135  140     * L          59 71          71B               EXT REFS  NOT INNER
  152  130       K          64 66           4B       INSTACK
  167          * K          67 67          11B               EXT REFS
  210          * K          70 70          11B               EXT REFS
  240  190     * I          76 92         103B               EXT REFS  NOT INNER
  245  180     * L          78 90          71B               EXT REFS  NOT INNER
  262  170       K          83 85           4B       INSTACK
  277          * K          86 86          11B               EXT REFS
  320          * K          89 89          11B               EXT REFS
  347  210     * L          94 97          15B               EXT REFS

STATISTICS
  PROGRAM LENGTH                 644B      420
       52000B CM USED
```



```
      PROGRAM TEST       73/74    OPT=1                        FTN 4.6+452        82/01/28. 14.34.48       PAGE     1

                      PROGRAM TEST(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)                     TEST          2
                C                                                                             LINE          2
                C---------------------------------------------------------------------------- LINE          3
                C                                                                             LINE          4
                C     TEST PROGRAM FOR WOLFOP                                                 TEST          4
                C                                                                             LINE          2
                C---------------------------------------------------------------------------- LINE          3
                C                                                                             LINE          4
                      DIMENSION Q(30,30), A1(10,30), A2(10,30), OBJ(50), IND(50)              TEST          6
                      DIMENSION SCR(3156), Y(30), IPASS(11)                                   TEST          7
                      LOGICAL FLAG                                                            TEST          8
                C                                                                             TEST          9
                C                                                                             LINE          2
                C---------------------------------------------------------------------------- LINE          3
                C                                                                             LINE          4
                      NAMELIST /IN/                                                           TEST         11
                     1  N, NCON, NUNC, NINEQ, NINV, FLAG, EPS,                                TEST         12
                     2  ITMAX, Q, A1, A2, OBJ,IND, C, VALUE                                   TEST         13
                C                                                                             LINE          2
                C---------------------------------------------------------------------------- LINE          3
                C                                                                             LINE          4
                   10 READ(5,IN)                                                              TEST         15
                      IF(EOF(5)) 999, 20                                                      TEST         16
                   20 NQD = 30                                                                TEST         17
                      N1D = 10                                                                TEST         18
                      N2D = 10                                                                TEST         19
                       IPASS(1) = N                                                           TEST         20
                      IPASS(2) = ITMAX                                                        TEST         21
                      IPASS(5) = NCON                                                         TEST         22
                      IPASS(6) = NUNC                                                         TEST         23
                      IPASS(7) = NINEQ                                                        TEST         24
                      IPASS(8) = NINV                                                         TEST         25
                      IPASS(9) = NQD                                                          TEST         26
                      IPASS(10) = N1D                                                         TEST         27
                      IPASS(11) = N2D                                                         TEST         28
                      CALL WOLFOP(Q,A1,A2,OBJ,IND,IPASS,C,VALUE,SCR,Y,FLAG,EPS)               TEST         29
                      CALL OPTAB(6,Q,A1,A2,OBJ,IND,IPASS,C,VALUE,Y,NQD,N1D,N2D)               TEST         30
                      GO TO 10                                                                TEST         31
                  999 CONTINUE                                                                TEST         32
                      STOP                                                                    TEST         33
                      END                                                                     TEST         34


     SYMBOLIC REFERENCE MAP (R=2)

 ENTRY POINTS      DEF LINE      REFERENCES
  4137  TEST           1

 VARIABLES      SN  TYPE            RELOCATION
  6121  A1          REAL        ARRAY            REFS        9          16          36          37
  6575  A2          REAL        ARRAY            REFS        9          16          36          37
  4310  C           REAL                         REFS       16          36          37
  4306  EPS         REAL                         REFS       16          36
  4300  FLAG        LOGICAL                      REFS       11          16          36
```

```
          PROGRAM TEST        73/74   OPT=1                        FTN 4.6+452       82/01/28. 14.34.48       PAGE     2

VARIABLES      SN  TYPE            RELOCATION
  7333  IND         INTEGER    ARRAY                   REFS         9         16         36         37
 15577  IPASS       INTEGER    ARRAY                   REFS        10         36         37     DEFINED        27         28         29
                                                         30        31         32         33         34         35
  4307  ITMAX       INTEGER                            REFS        16         28
  4301  N           INTEGER                            REFS        16         27
  4302  NCON        INTEGER                            REFS        16         29
  4304  NINEQ       INTEGER                            REFS        16         31
  4305  NINV        INTEGER                            REFS        16         32
  4312  NUD         INTEGER                            REFS        33         37     DEFINED        24
  4303  NUNC        INTEGER                            REFS        16         30
  4313  N1D         INTEGER                            REFS        34         37     DEFINED        25
  4314  N2D         INTEGER                            REFS        35         37     DEFINED        26
  7251  OBJ         REAL       ARRAY                   REFS         9         16         36         37
  4315  Q           REAL       ARRAY                   REFS         9         16         36         37
  7415  SCR         REAL       ARRAY                   REFS        10         36
  4311  VALUE       REAL                               REFS        16         36         37
 15541  Y           REAL       ARRAY                   REFS        10         36         37

FILE NAMES          MODE
     0  INPUT
  2054  OUTPUT
     0  TAPE5       NAME                      READS        22
  2054  TAPE6

EXTERNALS           TYPE   ARGS        REFERENCES
       EOF          REAL     1               23
       OPTAH                13               37
       WOLFOP               12               36

NAMELISTS           DEF LINE      REFERENCES
       IN                16          22

STATEMENT LABELS               DEF LINE    REFERENCES
  4141  10                         22          38
     0  20             INACTIVE    24          23
  4170  999                        39          23

STATISTICS
  PROGRAM LENGTH                 114628B    4914
  BUFFER LENGTH                   41308     2136
         52000B CM USED
```

APPENDIX B

MODIFICATION FOR PROJECTION PROBLEMS

WOLFQP as written will solve a projection problem. For large problems space may be reduced by modification to the program.

In the projection problem the Q matrix reduces to the identity. This need not be stored. The specific modifications are:

```
WOLFQP

   Delete the Do Loop with 130
   Replace with
  TEMP = TEMP + SCR(IP3 + I + N)

GETROW

   Delete Do Loop with 50
   Replace with
 AROW (J) = AROW(J) + E(KK + JJ)

PSOL

   Delete Do Loop with 40
   Replace with
      X(KI) = X(KI) + X(KK)
```

By then substituting a dummy argument for Q in the calling Sequence the user has removed the necessity of storing the identity matrix. Of course the user may then remove Q and NQD completely from the calling sequence if desired.

APPENDIX C

MACHINE DEPENDENCIES

BIGM - Defined by Data Statement in WOLFQP. BIGM represents machine infinity. For CDC Machines 10^{100} is used. This constant clearly depends on the exponent range available.

BASIC - An array used to store internal flags. To reduce space 7 bits of each BASIC word are used to store flags the remaining bits are required to store an integer. If the word size of the machine is M bits this integer must be less than $2^{M-7}-1$. For CDC machines this becomes $2^{53}-1$. This number $2^{M-7}-1$ represents the largest problem which can be solved with this encoding of the algorithm.

Since bit operations are performed on the Basic array certain bit functions must be utilized.

Octal constants are used to set certain bit patterns in SETQP, DSIMP and PIVROW. They are either used to set the bit patterns or mask certain bits from the given word. The octal constants appear only in Data Statements in the appropriate routines. They appear as integers followed by the letter B. These are numbers written in Base 8 and must be written with the appropriate notation and in the appropriate base for the machine.

i.e. 7B = 7 Hex
10B = 8 Hex
60B = 30 Hex

The Machine functions utilized are:

OR(A,B) - Perform bit by bit logical or on A,B. The truth table is

	0	1
0	0	1
1	1	1

AND(A,B) - Perform bit by bit logical and on A,B. The truth table is

	0	1
0	0	0
1	0	1

SHIFT(A,I) - SHIFT the word I bits. If I is positive SHIFT left I positions.

If I is negative shift right I positions.

i.e. if A contains the following bit pattern

A = 0001011010

then

SHIFT(A,2) = 0101101000
SHIFT(A,-2)= 0000010110

COMPL(A) - form bit by bit Boolean complement of A.

i.e. if A = 10110

COMPL(A)= 01001

The following library function is required in PIVCOL

RANF(X) - returns a uniform random number, the argument is ignored, in the function.
The random number is used in an anti-cycling procedure. Some other method of preventing cycling could be used instead.

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